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Monterey, California

Estimation of Parametric Length of Service Distributions of Advancees of the Navy Enlisted Force

by

Paul R. Milch

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A previous study using regression analysis established the relationship between the LOS distribution of advancements and the volume of advancements for three ratings of the Navy Enlisted Force. This relationship was the basis of a model to predict the number of advancements by LOS in future years. The present study used that (regression) model as a starting point to accomplish the same goal with a gamma distribution model appropriately parametrized to make it dependent on volume of advancements. This latter model appears nearly

as accurate in its predictions as the regression model, while reducing the number of parameters from ninety-three to fifteen for each pay grade of each rating. The relationship between mean LOS and volume of advancements remains approximately the same as in the case of the regression model.

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FOREWORD

This research effort was initiated by the Naval Personnel Research and Development Center (NPRDC), San Diego, California and subsequently sponsored by the Bureau of Naval Personnel (BUPERS), Washington, D.C. The author would like to express his appreciation for the valuable assistance provided by the staffs of Mr. Robert K. Lehto of BUPERS and Mr. Joe Silverman of NPRDC.

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I. INTRODUCTION

A. Problem Statement

Advancements to the top six pay grades of the Navy Enlisted Force are brought about through a centrally managed system (Silverman, 1977). This process basically consists of promotions to fill vacancies created by requirements and limited by available resources. However, it is further constrained by promotion policy rules, D.O.D. restrictions, etc. A computer model, FAST (Boller, 1974; Silverman, 1977), used by the Bureau of Naval Personnel simulates (in a non-statistical way) many of the features of the advancement system. An interactive computer model, called MINIFAST (Butterworth, 1976), was recently developed at the Naval Postgraduate School. These models are used to make future predictions of end strengths of the Navy in great detail. They are also used to answer policy questions posed by personnel managers. Analysis is required for specific ratings and pay grades in terms of length of service (LOS). Both FAST and MINIFAST fall short on making accurate predictions of advancements by LOS. The present method involves estimating rates of advancements from historical records (Leland, 1976); then both models use these rates together with the LOS distribution of the resource population to make estimates of the number of promotions to a pay grade by LOS. One particular shortcoming of this method is its insensitivity to changes in the total number (volume) of advancements to a pay grade. In the actual planning cycle, these numbers (i.e. the pay grade totals of advancements) are computed first and the LOS distribution of advancements becomes quite volatile as the volume varies.

It is the objective of this study to make a contribution toward the accurate prediction of the LOS distribution of advancements. In particular, it is desirable to make such a procedure dependent explicitly on the volume of advancements to the pay grade in question. Such a procedure, if successful, has the potential of being incorporated in FAST and/or MINIFAST.

Aside from these models, other immediate applications also suggest themselves. Various models used in personnel planning by BUPERS are forced to make rather arbitrary, however reasonable, assumptions concerning the behavior of the advancement system. It is expected that the specific analytical forms described in this report may be of use in other existing personnel planning models.

B. Background

In a previous report (Milch, 1976) a regression model was constructed to predict the LOS distribution of advancements. This model had the advantage of being dependent on the volume of advancements to a pay grade and appeared reasonably accurate when predictions were made for years for which actual advancement data was available. The model is a simple regression of advancements on resources and the volume of advancements. The feasibility of its being incorporated in FAST is now being determined.

A drawback of this regression model stems from the fact that the regression is carried out in each of the thirty-one LOS cells separately. This results in ninety-three parameter values for each LOS distribution of advancements to a pay grade. This may not overburden a vast computer model such as FAST, but is certainly not feasible for the simpler and smaller MINI-FAST. For this and other reasons, a more compact model was sought that would rely more on analytical tools to predict advancements while retaining the volume dependent feature of the regression model.

C. Data Source

The data available for this study was originally that available for the Regression Model (Milch, 1976). This data was based on the Pay Entry Base Date (PEBD) accounting of the length of service of Naval Personnel and covered the period 1966-74. As the FY 1975 and 1976 data became available during the course of this study, the definition of LOS was switched to the Total Active Federal Military Service (TAFMS). In order to make full use of the data for the entire 1966-76 period all computation was repeated with both advancement and inventory data in the TAFMS format.

Another change from the previous study (Milch, 1976) was the use of net inventories in place of (beginning) inventories. Net inventories are defined to be the beginning inventory less losses plus non-recruit gains to the Navy during the year. This inventory, defined for every rating, pay grade, and length of service cell, is conceptually an estimate of the total resources available to the advancement system. Of course, not everyone in the net inventory is generally eligible for promotions, however the net inventory can be expected to be more closely correlated to actual advancements than beginning inventory.

In other respects the data had the same characteristics as that used for the previous study. Thus, the ratings used were 300, 1500, 1800 and 0. The latter stands for total Navy, usually labeled as ALLNAVY. Only advancements to the top six pay grades were considered. Since the inventory in the next lower pay grade is the appropriate resource population for advancements to a pay grade, the pay grades of inventory used were E3-E8, except that E3 actually contained the total personnel in the three lowest pay grades.

II. PROBLEM DEVELOPMENT

A. Statistical Formulation of the Problem

In order to develop an analytic solution to the problem of predicting advancements, the problem itself will be specified in precise analytic terms.

The discussion in this section will refer to advancements to a specific pay grade in a specific rating. For this reason the notation of pay grade and rating will be suppressed. Similarly the period or fiscal year involved will be made explicit in the notation only when needed. Let

V = total number of advancees to a pay grade in a rating during the fiscal year.

These V advancements are, of course, effected at various times during the fiscal year. In that sense, some ordering among them could be established. It is, however, not in that sense that subsequent mention will be made of the first, second or, in general, the "j^{th} advancee." Instead, a conceptual ordering is imagined among advancees for the sake of mathematical exposition. (For example, one may think of alphabetical ordering of the surnames of advancees.) In this sense, let, for $j = 1,2,\ldots,V$,

$$L_j = \text{the LOS year of the } j^{th} \text{ advancee.}$$

The most important thing to note about the quantities L_j $(1 \leq j \leq V)$ is that their values are not available. Instead, aggregate numbers of advancements, i.e., the quantities:

$$A_i = \text{number of advancements in LOS cell } i, \text{ for } i = 1,2,\ldots,31,$$

were available in existing data files.

Briefly stated, the problem of predicting future advancements is one of estimating the distribution of L_j from data on the variables A_i, $1 \leq i \leq 31$. To make this idea more precise some further notation is necessary.

The conditional distribution of L_j given the volume of advancements is introduced:

$$f_i(V) = P(L_j = i|V) \tag{1}$$

independent of j for $i = 1,2,\ldots,31$. Also, the conditional mean and variance of L_j are denoted by

$$\mu_L(V) = E(L_j|V) = \sum_{i=1}^{31} i f_i(V) \tag{2}$$

and

$$\sigma_L^2(V) = \mathrm{Var}(L_j|V) = E(L_j^2|V) - (E(L_j|V))^2 \tag{3}$$

As the notation implies the random variables L_j, $1 \le j \le V$, are identically distributed when V is given. It can also be argued that they are independently distributed, when V is given, because the ordering of advancements is randomized rather than chronological.

In mathematical terms the precise assumption, supported by practical considerations, is that when V is given the random variables $L_1,\ldots,L_V$ are independently and identically distributed with the parameters given in equations (1), (2), and (3) above.

It is precisely this conditional distribution of L_j, given V, that is needed if future predictions of advancements are to be made based on some values of V that are either arbitrarily selected by personnel managers or are forced on them by the circumstances.

The data available for this study concerned not the variables L_j, $1 \le j \le V$ but the variables A_i, $1 \le i \le 31$. Observations on A_i, $1 \le i \le 31$, have been available for the fiscal years 1966-1976. This data will be referred to by the notation:

$$A_1^{(k)},\ldots,A_{31}^{(k)} \tag{4}$$

for $k = 1,\ldots,11$, where "$k = 1$" means FY 1966 and "$k = 11$" refers to FY 1976.

The problem may now be stated precisely as one of estimating the conditional distribution (1) of L_j, given V, or at least its mean (2) and variance (3), from the observations (4) on the random variables A_i, $1 \le i \le 31$. By necessity, this distribution and its estimate must be a function of V.

B. The Relationship Between the Distributions of L_j and A_i

In order to estimate the conditional distribution of L_j, given V, from observations on the variables A_i, $1 \leq i \leq 31$, first the relationship between the distributions of these random variables must be established. Most of these results are intuitively obvious and simple to prove. They are explained here because their exposition is thought to contribute to clarifying the problem under discussion.

The relationship between the random variables L_j, $1 \leq j \leq V$, and A_i, $1 \leq i \leq 31$ may best be established formally through the use of the auxiliary random variables:

$$U_{ij} = \begin{cases} 1 \text{ if } L_j = i \\ 0 \text{ otherwise} \end{cases}$$

$$\text{for } i = 1,\ldots,31 \text{ and } j = 1,\ldots,V.$$

In words, U_{ij} is defined as an indicator variable whose value is one if the j^{th} advancee happens to be in the i^{th} LOS cell. The following relationships hold:

$$\sum_{i=1}^{31} U_{ij} = 1, \text{ for } 1 \leq j \leq V \tag{5}$$

$$\sum_{j=1}^{V} U_{ij} = A_i, \text{ for } 1 \leq i \leq 31 \tag{6}$$

$$\sum_{i=1}^{31} g(i)U_{ij} = g(L_j), \text{ for } 1 \leq j \leq V \text{ and any function } g(i). \tag{7}$$

Formula (5) expresses the fact that any advancee is in one and only one LOS cell. Formula (6) explains that for a specific LOS cell one way to count advancees is simply to add up all the indicator variables for that cell. Formula (7) is easy to see first in its simplest form:

$$\sum_{i=1}^{31} iU_{ij} = L_j, \text{ for } 1 \leq j \leq V, \tag{8}$$

which is true precisely because $iU_{ij} = L_j$ when $L_j = i$ and $iU_{ij} = 0$ otherwise; the more general form of (7) holds for essentially the same reason; another form of (7) that will be used is, for any r,

$$\sum_{i=1}^{31} i^r U_{ij} = L_j^r, \text{ for } 1 \leq j \leq V \quad . \tag{9}$$

Another relationship connecting the variables V and A_i, $1 \leq i \leq 31$, is:

$$\sum_{i=1}^{31} A_i = V \, .$$

From the assumption that the variables $L_1, \ldots, L_V$ are independent and identically distributed random variables when V is given, the same statement obviously follows for the variables $U_{i1}, \ldots, U_{iV}$ for any fixed $i = 1, \ldots, 31$.

The conditional distribution of U_{ij}, given V, is clearly given by the next statement.

<u>Statement 1</u>.

U_{ij} is a Bernoulli random variable with parameter $f_i(V)$ independent of j when V is given. Thus, the conditional mean and variance are

$$E(U_{ij} | V) = f_i(V) \tag{10}$$

$$\mathrm{Var}(U_{ij} | V) = f_i(V)[1 - f_i(V)] \tag{11}$$

for $1 \leq i \leq 31$ and $1 \leq j \leq V$.

It is intuitively appealing to use the ratio of A_i to V when attempting to estimate the theoretical probabilities $f_i(V)$, that an arbitrary advancee is in LOS cell i. To explore this idea, the variable

$$F_i = \frac{A_i}{V} \, , \qquad \text{for } i = 1, \ldots, 31$$

is introduced.

Then the next statement follows.

Statement 2.

The conditional mean and variance of F_i, when V is given, are, for $1 \leq i \leq 31$,

$$E(F_i | V) = f_i(V) \tag{12}$$

and

$$Var(F_i | V) = \frac{1}{V} f_i(V)[1 - f_i(V)] \ . \tag{13}$$

Proof of these formulas is quite simple and is given in Appendix A.

Statement 2 explains why empirical values of F_i are commonly used to estimate (or even confused with) the theoretical concept of the probability, $f_i(V)$, that an advancee is in LOS cell i. Indeed, F_i is an unbiased estimate of this probability and for high values of V it is an estimate with small variance. This fact was exploited (even if not explictly stated) in the regression analysis reported previously (Milch, 1976).

In the current study, however, the estimation of the (conditional) distribution of L_j (given V) will be approached through its (conditional) moments. For this reason, the following empirical moments of F_i are introduced:

$$K_r = \sum_{i=1}^{31} i^r F_i \qquad \text{for any r.} \tag{14}$$

In particular, the first and second moments will be used to estimate the theoretical conditional moments of L_j when V is given. That this may be successfully accomplished is suggested by the following.

Statement 3.

$$K_r = \frac{1}{V} \sum_{j=1}^{31} L_j^r \qquad \text{for } 1 \leq j \leq V \text{ and any r.} \tag{15}$$

The short proof is in Appendix A.

Formula (15) shows that, for any r, K_r is the r^{th} sample moment of the observations L_j, $1 \leq j \leq V$, when V is given. Also,

$$K_2' = \frac{V}{V-1} \sum_{i=1}^{31} (i - K_1)^2 F_i = \frac{V}{V-1} (K_2 - K_1^2) \tag{16}$$

is equal to the sample variance of L_j, $1 \leq j \leq V$, when V is given, i.e.,

$$K_2' = \frac{1}{V-1} \sum_{i=1}^{31} (L_j - K_1)^2 \tag{17}$$

From the above it follows

Statement 3.

When V is given K_1 and K_2' are unbiased estimates of the conditional mean, $\mu_L(V)$, and variance, $\sigma_L^2(V)$, of L_j, with conditional variances

$$\mathrm{Var}(K_1|V) = \frac{1}{V} \sigma_L^2(V)$$

and

$$\mathrm{Var}(K_2'|V) = \frac{1}{V}[\mu_L^{(4)}(V) - \frac{V-3}{V-1} \sigma_L^4(V)]$$

where

$$\mu_L^{(4)}(V) = E(L_j^4|V)$$

is the fourth conditional moment of L_j, given V.

These are well known results of statistics (see e.g. Wilks, 1962, pages 199-200).

These results imply that K_1 and K_2' are the traditionally used statistical estimates of the mean and variance of L_j and could be used to estimate the distribution of L_j as well.

C. The Regression Model

In a previous study (Milch, 1976) two regression models were designed. The purpose of these models was to estimate the LOS distribution of advancements as a function of the pay grade total (volume) of advancements and the LOS distribution of some suitably chosen resource population. The model that proved to be the more practicable of the two used (beginning) inventories as the resource population for advancements. This model is mathematically described by the equations:

$$\hat{A}_i = V \frac{[\alpha_i + \beta_i I_i + \gamma_i V]^+}{\sum_{i=1}^{31} [\alpha_i + \beta_i I_i + \gamma_i V]^+}, \qquad \text{for } i = 1,\ldots,31, \tag{18}$$

where

$\hat{A}_i$ = estimated number of advancees in LOS cell i;

I_i = (beginning) inventories in LOS cell i of the originating pay grade;

$[x]^+$ = max $(0,x)$;

and the coefficients α_i, β_i, γ_i are the results of the regression analysis. As equation (18) shows the results of the regression analysis were altered to eliminate negative numbers and renormalized in order that advancements in all LOS cell sum to the "correct" pay grade total of advancements, V. Then the LOS distribution of advancements, as estimated by the regression model, is

$$\hat{F}_i = \frac{\hat{A}_i}{V} = \frac{[\alpha_i + \beta_i I_i + \gamma_i V]^+}{\sum_{i=1}^{31} [\alpha_i + \beta_i I_i + \gamma_i V]^+}, \qquad i = 1,\ldots,31 \tag{19}$$

In the previous report (Milch, 1976) it was shown that this distribution of advancements was reasonbly accurate. This was accomplished by computing these $\hat{F}_i$, $1 \leq i \leq 31$, values for several of the FY's for which data was available and then comparing them to the actual $F_i = A_i/V$, $1 \leq i \leq 31$, values. The volume dependent behavior of the LOS distribution of advancements was also exhibited and explained by this model.

However, it may be noted that this regression model necessitates the computation, storage and use of 3 x 31 = 93 parameter values (the α_i's, β_i's and γ_i's) for the prediction of advancements to any one pay grade and rating. Also, the prediction mode makes use of the LOS distribution (i.e., 31 numbers) of inventories (or some other surrogate for advancement resources) in the originating pay grade.

In order to make the model more adaptable to certain personnel management functions it appeared desirable to reduce significantly the number of parameters on which it depends.

D. The Moments of the Regression Model

As was explained in the previous section the estimates $\hat{F}_i$, as given by Formula (19), were reasonably good approximations of the sample distribution $F_i = A_i/V$ and therefore may be regarded as estimates of the theoretical distribution, $f_i(V)$, of L_j'. The same procedure is suggested for the moments as well. Since K_1 and K_2', as defined by Formulas (14) and (16) are unbiased estimates of $\mu_L(V)$ and $\sigma_L^2(V)$ resp., it may be expected that the corresponding moments defined in terms of $\hat{F}_i$ will serve as "good" estimates as well. The following quantities are defined:

$$\hat{K}_r = \sum_{i=1}^{31} i^r \hat{F}_i, \qquad \text{for any } r \ . \tag{20}$$

In the previous report (Milch, 1976) the mean, $\hat{K}_1$, and the variance $\hat{K}_2 - \hat{K}_1^2$, were compared to the corresponding quantities computed from the data. These figures were displayed together with the "predictions" of advancements for past years. Reasonably good agreement was found. In order to compute these moments, however, it is necessary to obtain the 93 parameter values referred to at the end of Section C.

It is possible, however, to approximate the expressions (20) with quantities whose computation requires fewer parameter values than indicated above. First, the function $[\ \ ]^+$ will be disregarded in Formula (19) and $\hat{K}_r$ computed accordingly from (20). The justification for neglecting $[\ \ ]^+$ is empirical: there were relatively few instances when the regression analysis resulted in negative number of advancements; clearly, these cases occurred mostly in very sparsely populated LOS cells. The resulting approximation for the moments is, for any r,

$$\hat{K}_r \approx \frac{\sum_{i=1}^{31} i^r \alpha_i + \sum_{i=1}^{31} i^r \beta_i I_i + V \sum_{i=1}^{31} i^r \gamma_i}{\sum_{i=1}^{31} \alpha_i + \sum_{i=1}^{31} \beta_i I_i + V \sum_{i=1}^{31} \gamma_i} \ . \tag{21}$$

It may be noted at this point that this formula of $\hat{K}_r$ involves only two functions of the thirty-one α_i values: $\sum\alpha_i$ and $\sum i^r\alpha_i$. The same holds true for the γ_i values. A similar statement may not be made about the β_i's: all thirty-one β_i values are needed to recompute $\sum\beta_i I_i$ and $\sum i^r\beta_i I_i$ as the inventory distribution takes on different forms. It is the subject of the next section to obtain a reduction in the number of β_i parameters needed to approximate $\hat{K}_r$.

E. Approximation of the Moments

To make future explanations easier the following notation is introduced:

$$S_r(\alpha) = \frac{1}{n_r} \sum_{i=1}^{31} i^r \alpha_i \qquad \text{for any } r \tag{22}$$

$$\text{where} \quad n_r = \sum_{i=1}^{31} i^r \quad . \tag{23}$$

With this notation the r^{th} moment $\hat{K}_r$, may be written, for any r,

$$\hat{K}_r \approx \frac{n_r}{n_o} \frac{S_r(\alpha) + S_r(\beta I) + S_r(\gamma)V}{S_o(\alpha) + S_o(\beta I) + S_o(\gamma)V} \tag{24}$$

where $S_r(\beta I)$ and $S_r(\gamma)$ are defined analogously to $S_r(\alpha)$ in (22). Although (22), (23) and (24) are defined for any r, the values of immediate interest are r = 0, 1, 2. Note that

$$n_o = \sum_{i=1}^{31} 1 = 31, \quad n_1 = \sum_{i=1}^{31} i = \frac{(31)(32)}{2} = 496, \quad n_2 = \sum_{i=1}^{31} i^2 = \frac{(31)(32)(63)}{6} = 10{,}416.$$

The formulas for the first and second estimated moments are, in particular,

$$\hat{K}_1 \simeq 16 \frac{S_1(\alpha) + S_1(\beta I) + S_1(\gamma)V}{S_o(\alpha) + S_o(\beta I) + S_o(\gamma)V} \tag{25}$$

and

$$K_2 \simeq 336 \frac{S_2(\alpha) + S_2(\beta I) + S_2(\gamma)V}{S_o(\alpha) + S_o(\beta I) + S_o(\gamma)V} \quad . \tag{26}$$

The immediate goal is to permit the computation (or approximation) of these moments without the full use of the thirty β_i values. This will be accomplished by approximating $S_r(\beta I)$, for r = 0, 1, 2, although the procedure will be explained in terms of arbitrary r.

The fact that $S_o(\beta I)$ (and in a more complicated way $S_r(\beta I)$, for any r, as well) is the sum of products of thirty-one pairs, (β_i, I_i), of numbers suggests that this quantity is related to covariances and correlations.

To make this remark more precise pairs of new random variables will be introduced whose purpose is purely conceptual and exact meaning is of no importance to the original problem.

For any r, the pair $(\mathcal{B}_r, \mathcal{I}_r)$ of random variables is defined to have the joint distribution

$$P(\mathcal{B}_r = \beta_i, \mathcal{I}_r = I_j) = \begin{cases} \dfrac{i^r}{n_r} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

for i, j = 1,...,31.

For technical reasons, this definition is not precise, unless the values β_i, $1 \leq \beta \leq 31$ as well as the values I_i, $1 \leq i \leq 31$ are all distinct. Although often this is not the case, the difficulty is merely a technical one and will be ignored in this section. In Appendix B additional explanation is given that overcomes this problem at the expense of increased notational complexity.

Formula (23) assures that (27) defines a joint probability mass function. The marginal probabilities are

$$P(\mathcal{B}_r = \beta_i) = P(\mathcal{I}_r = I_i) = \frac{i^r}{n_r} \qquad \text{for } i = 1,\ldots,31 \quad . \tag{28}$$

The marginal and joint moments are computed as

$$E(\mathcal{B}_r^m) = \sum_{i=1}^{31} \beta_i^m \frac{i^r}{n_r} = S_r(\beta^m) \qquad \text{for any } m \quad ;$$

$$E(\mathcal{I}_r^n) = \sum_{i=1}^{31} I_i^n \frac{i^r}{n_r} = S_r(I^n) \qquad \text{for any } n \quad ;$$

$$E(\mathcal{B}_r^m \mathcal{I}_r^n) = \sum_{i=1}^{31} \beta_i^m I_i^n \frac{i^r}{n_r} = S_r(\beta^m I^n) \text{ for any } m \text{ and } n \quad . \tag{29}$$

The following means, variances and covariance are of immediate interest:

$$E(\mathcal{B}_r) = S_r(\beta) \tag{30}$$

$$E(\mathcal{I}_r) = S_r(I) \tag{31}$$

$$\mathrm{Var}(\mathcal{B}_r) = S_r(\beta^2) - S_r^2(\beta) \tag{32}$$

$$\mathrm{Var}(I_r) = S_r(I^2) - S_r^2(I) \tag{33}$$

$$\mathrm{Cov}(B_r,I_r) = S_r(\beta I) - S_r(\beta)S_r(I) \tag{34}$$

Formula (34) suggests a way of approximating $S_r(\beta I)$. The correlation coefficient of the joint random variables B_r and I_r is defined as

$$\rho_r = \rho(B,I_r) = \frac{\mathrm{Cov}(B_r,I_r)}{\sqrt{\mathrm{Var}(B_r)\mathrm{Var}(I_r)}} \quad . \tag{35}$$

It may be recalled that the correlation coefficient of two random variables lies always betwen -1 and +1 and as such its sample equivalent (i.e., the sample correlation coefficient frequently used to estimate the "population" correlation coefficient) must be much more stable than the corresponding sample covariance or mixed moment. It is, therefore, suggested to put Formulas (30) - (35) together to express $S_r(\beta I)$ in terms of the means, variances and correlation coefficients of the joint random variables (B_r,I_r):

$$S_r(\beta I) = S_r(\beta)S_r(I) + \rho_r \sqrt{[S_r(\beta^2)-S_r^2(\beta)] \quad [S_r(I^2)-S_r^2(I)]} \quad . \tag{36}$$

The difficulty with using (36) as written is, of course, the circular nature of the definitions given above. The correlation coefficients, ρ_r, were defined in terms of $S_r(\beta I)$ and hence (36) does not, in itself, provide a way to calculate $S_r(\beta I)$. However, there is eleven years of data (for FY's 1966-76) available to compute sample estimates of ρ_r. The β_i, $1 \leq i \leq 31$, values remain unchanged as computed from the regression analysis. The I_i, $1 \leq i \leq 31$, values are available for each of the eleven years. This provides eleven estimates for each ρ_r computed in accordance with Formula (35). These values for r = 0, 1, 2 are listed in Appendix C for each of the six pay grades E4 through E9 and ratings 300, 1500, 1800 and 0 (ALLNAVY). The numbers display a reasonably good stability over the ten year period. It is also noteworthy that each pay grade has its own particular range of values.

There are several ways in which the eleven data values available for ρ_r, $r = 0, 1, 2$, may be used. Some sort of average should be used for reasons of stability, but it also seemed that early years would bear less relevance for future planning than values obtained from more recent years. Finally, it was decided, somewhat arbitrarily, to use the average of the ρ_r values obtained from the last five years' (FY's 1972-76) data. These values are denoted by $\hat{\rho}_r$ (as estimates of ρ_r) for $r = 0, 1, 2$, and are given in Appendix C for all pay grades and ratings.

Therefore, the following quantities may be used as approximations of the $S_r(\beta I)$ values:

$$\hat{S}_r(\beta I) = S_r(\beta)S_r(I) + \hat{\rho}\sqrt{[S_r(\beta^2) - S_r^2(\beta)]\,[S_r(I^2) - S_r^2(I)]} \quad . \tag{37}$$

An approximating formula for the r^{th} estimated moment K_r is, for any r,

$$\hat{K}_r \simeq \frac{n_r}{n_o}\,\frac{S_r(\alpha) + \hat{S}_r(\beta I) + S_r(\gamma)V}{S_o(\alpha) + \hat{S}_o(\beta I) + S_o(\gamma)V} \quad , \tag{38}$$

where $\hat{S}_r(\beta I)$ and $\hat{S}_o(\beta I)$ are given by (37).

It may be noted immediately that to compute Formula (38) neither the thirty-one β_i values nor the thirty-one inventories I_i are needed directly. Instead, the six quantities: $S_o(\beta)$, $S_o(\beta^2)$, $S_r(\beta)$, $S_r(\beta^2)$, $\hat{\rho}_o$ and $\hat{\rho}_r$ replace β_i $1 \leq i \leq 31$. Similarly, the four quantities: $S_o(I)$, $S_o(I^2)$, $S_r(I)$ and $S_r(I^2)$ replace I_i, $1 \leq i \leq 31$.

For purposes of estimating the conditional LOS distributions $f_i(V)$, $1 \leq i \leq V$, of L_j when V is given it will be necessary to use the first and second moments $\hat{K}_1$ and $\hat{K}_2$. These may now be computed from Formula (38) with $r = 1$ and 2. The parameters needed to compute these moments are: $S_r(\alpha)$,

$S_r(\gamma)$, $S_r(\beta)$, $S_r(\beta^2)$ and $\hat{\rho}_r$ for $r = 0, 1, 2$. These fifteen parameters replace the $3 \times 31 = 93$ parameter values of α_i, β_i, γ_i for $1 \le i \le 31$. Although these fifteen quantities must originally be computed from the ninety-three regression coefficients, they may be computed "once and for all" and used repeatedly with various volume levels and inventory distributions. An additional simplification is that the thirty-one inventory, I_i, $1 \le i \le 31$, values are also replaced by six quantities: $S_r(I)$ and $S_r(I^2)$ for $r = 0, 1, 2$.

F. Estimation of the LOS Distribution of Advancements.

With the approximation of the moments, $\hat{K}_r$, by relatively simple expressions that exhibit the dependence on the volume of advancements, the estimation of parameters for the distribution of advancements is also feasible.

This is achieved by the following procedure.

1. The Regression Analysis described previously (Milch, 1976 and in Section II.C. of this report) is recomputed with I_i redefined as net inventories, equal to beginning inventories less losses plus non-recruit gains (see Section I.C.). The result of this computation is the set of ninety-three coefficients: α_i, β_i and γ_i, for $1 \leq i \leq 31$.

2. Using Formula (38) the first and second sample moments are computed for a given volume V and inventory distribution, I_i, $1 \leq i \leq 31$. In particular:

$$\hat{K}_1 = 16 \frac{S_1(\alpha) + \hat{S}_1(\beta I) + S_1(\gamma)V}{S_o(\alpha) + \hat{S}_o(\beta I) + S_o(\gamma)V} \tag{39}$$

and

$$\hat{K}_2 = 336 \frac{S_2(\alpha) + \hat{S}_2(\beta I) + S_2(\gamma)V}{S_o(\alpha) + \hat{S}_o(\beta I) + S_o(\gamma)V} \tag{40}$$

where $S_r(\alpha)$ and $S_r(\gamma)$, for $r = 0,1,2$, are defined by Formula (22) and $\hat{S}_r(\beta I)$, for $r = 0,1,2$, is given by Formula (37). Next the sample variance is computed from

$$\hat{K}_2' = \frac{V}{V-1} (\hat{K}_2 - K_1^2) \quad . \tag{41}$$

3. Using the sample mean and variance the two parameters, g and λ, of the gamma distribution, with density function

$$f(i;\, g,\, \lambda) = \lambda \frac{(\lambda i)^{g-1}}{\Gamma(g)} e^{-\lambda i} \quad , \quad i \geq 0 \quad , \tag{42}$$

are estimated via the method of moments.

That is, the estimates of g and λ are

$$\hat{g} = \frac{\hat{K}_1^2}{\hat{K}_2'} \quad \text{and} \quad \hat{\lambda} = \frac{\hat{K}_1}{\hat{K}_2'} \quad . \tag{43}$$

4. Using these estimates of the parameters, g and λ, the density function (42) is computed for integer values $i = 1,2,\ldots,31$ and renormalized to assure it defines a distribution when used at these values. The ensuing probablity function

$$\hat{f}(i;\, \hat{g},\, \hat{\lambda}) = \frac{f(i;\, \hat{g},\, \hat{\lambda})}{\sum_{j=1}^{31} f(j;\, \hat{g},\, \hat{\lambda})}, \quad i = 1,\ldots,31, \tag{44}$$

is used as the estimated LOS distribution of advancements.

5. The $\hat{f}(i;\, \hat{g},\, \hat{\lambda})$ values are multiplied by the volume, V , of advancements and rounded to the nearest integer to provide estimates of the number of advancees to a pay grade by LOS.

In order to display the result of this procedure, these estimated advancees, by LOS, to a pay grade are compared to the actual number of advancements, the estimates provided by the Regression Model and the estimates as computed by current FAST methodology. The comparison for FY 1976 is shown graphically in Appendix D for ratings 300, 1500, 1800 and 0 (ALLNAVY) for advancements to the six upper pay grades. The FAST methodology to which reference was made above involves the computation of historical rates of advancements described in an NPRDC working paper (Leland, 1976). If these rates are denoted by H_i for $1 \leq i \leq 31$, the advancement LOS distribution is given by the formula

$$F_i' = \frac{.9H_i + .1I_i}{\sum_{j=1}^{31} (.9H_j + .1I_j)} \qquad \text{for } 1 \leq i \leq 31 \tag{45}$$

where I_i denotes the net inventory in LOS cell i and the next lower pay grade.

The gamma distribution was chosen after several other candidates such as the beta, binomial, negative binomial, Weibull, lognormal and other distributions were also considered. Occasionally the estimate of the shape parameter, $\hat{g}$, of the gamma distribution is too large for easy computation of the probabilities $\hat{f}(i; \hat{g}, \hat{\lambda})$. In such cases the normal distribution with density function

$$f(i; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(i-\mu)^2}{2\sigma^2}}$$

is used in step 3. above. The parameters μ and σ^2 are estimated by

$$\hat{\mu} = \hat{K}_1 \quad \text{and} \quad \hat{\sigma}^2 = \hat{K}_2'$$

and then the renormalized probability function $\hat{f}(i; \hat{\mu}, \hat{\sigma}^2)$, defined analogously to Formula (44), is used for $1 \leq i \leq 31$ in steps 4. and 5. above. This procedure was used in rare cases when $\hat{g} > 100$.

G. Evaluation of the Estimation Procedure.

The graphs in Appendix D reveal that in most cases the estimates provided by the gamma distribution do better than those of the FAST method and almost as well as the regression estimates. It is not easy, however, to compare quantitatively these three estimates from these graphs. For this reason, three measures of error were constructed and the three estimates compared in terms of these measures. The three measures are:

1. The difference between the actual and estimated mean LOS value:

$$\Delta_1 = K_1 - \tilde{K}_1$$

where K_1 is the actual mean LOS of advancements given by Formula (14) and $\tilde{K}_1$ denotes the mean LOS of one of the three estimates of advancements. For the estimate given by the gamma distribution $\tilde{K}_1 = \hat{K}_1$ as defined by Formula (39). For the regression and FAST models the means, $\tilde{K}_1$, are computed in the usual way from the distribution.

2. The difference in standard errors: This is the standard deviation of the actual LOS distribution less the standard deviation from the actual mean LOS of the estimated LOS distribution:

$$\Delta_2 = \sqrt{K_2'} - \sqrt{\tilde{K}_2} \quad .$$

Here K_2' is the sample variance of the actual LOS distribution of advancements, given by Formula (16) and $\tilde{K}_2$ is the sum of squares of the differences between the estimated number of advancees and the actual mean LOS of advancements. For example, for the gamma distribution estimate $\tilde{K}_2 = \hat{K}_2''$ where $\hat{K}_2''$ is defined as

$$\hat{K}_2'' = \frac{V}{V-1} \sum_{i=1}^{V} (i - K_1)^2 \hat{f}_i = \frac{V}{V-1}(\hat{K}_2 - 2K_1\hat{K}_1 + K_1^2) \quad .$$

Note that $\hat{K}_2''$ differs from $\hat{K}_2'$, as defined by (41), and its square root may be appropriately called the standard error of the estimate. $\tilde{K}_2$ is computed analogously for the Regression and FAST models.

3. The Kolmogrov-Smirnov (K-S) Statistic: This is the largest absolute difference between the actual and estimated cumulative sample distribution functions:

$$\Delta_3 = \max_{1 \le i \le 31} \left| \sum_{j=1}^{i} F_j - \sum_{j=1}^{i} \tilde{F}_j \right| ,$$

where F_j is the actual relative frequency of advancees in LOS cell j and $\tilde{F}_j$ is the corresponding estimate. In particular,

$$\tilde{F}_j = \begin{cases} \hat{f}(j;\ \hat{g},\ \hat{\lambda}) & \text{for the gamma distribution model} \\ \hat{F}_j & \text{for the Regression model} \\ F_j' & \text{for the FAST model} \end{cases}$$

where the quantities on the righ hand side are given by Formulas (44), (19), and (45), respectively.

The values obtained for these three measures when estimates for FY 1976 were made are displayed in Table 1 below for all pay grades and ratings considered. The figures in this table confirm what was tentatively concluded after observing the graphs of the estimated numbers of advancements in Appendix D. Pay grade E5 of rating 300 appears to be the only significant case where the FAST methodology is superior. In some cases the improvement provided by the gamma distribution over FAST is quite significant; see e.g. pay grade E6 of rating 1500.

To compare errors for other FY's Appendix E shows graphs of the errors Δ_1 and Δ_3 against FY's 1966-76. These graphs, by and large, confirm the findings above. In fairness to the FAST methodology it must be pointed out that the estimates obtained by it here used Formula (44) with the same H_i, $1 \le i \le 31$, values for every FY, even though in practice new historically estimated rates are computed for every FY.

TABLE 1

COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS

RATING=300 *YEAR*=1976

ACTUAL - ESTIMATED MEAN LOS OF ADVANCEMENTS:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.02	0.23	0.17	0.24	¯0.63	0.50
GAMMA DIST	0.16	0.43	¯0.14	0.42	¯0.38	0.50
F A S T	0.64	0.23	0.62	1.95	¯0.52	0.30

ACTUAL ST. DEV. - ST. ERROR OF ESTIMATE OF LOS DIST.:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.17	¯0.58	0.16	0.24	0.25	¯0.21
GAMMA DIST	0.41	¯0.54	¯0.01	¯0.20	0.15	¯0.81
F A S T	0.12	¯0.94	0.25	¯1.47	0.12	¯1.08

K-S STATISTIC:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.03	0.22	0.06	0.07	0.22	0.33
GAMMA DIST	0.05	0.34	0.13	0.09	0.18	0.27
F A S T	0.37	0.17	0.15	0.25	0.19	0.27

RATING=1500 *YEAR*=1976

ACTUAL - ESTIMATED MEAN LOS OF ADVANCEMENTS:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.02	0.17	1.11	¯0.19	¯0.80	¯0.10
GAMMA DIST	0.10	0.70	0.42	¯0.67	¯0.79	¯0.10
F A S T	0.56	0.97	2.39	¯0.29	¯1.22	¯0.52

ACTUAL ST. DEV. - ST. ERROR OF ESTIMATE OF LOS DIST.:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.16	0.02	¯0.73	¯0.08	1.10	0.09
GAMMA DIST	0.32	0.17	¯0.47	0.50	1.15	¯1.04
F A S T	0.25	¯0.38	¯1.07	¯0.11	0.28	¯1.03

K-S STATISTIC:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.03	0.07	0.25	0.06	0.13	0.08
GAMMA DIST	0.06	0.26	0.11	0.14	0.13	0.14
F A S T	0.28	0.39	0.43	0.08	0.13	0.12

TABLE 1 (cont'd)

COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS

RATING=1800 *YEAR*=1976

ACTUAL - ESTIMATED MEAN LOS OF ADVANCEMENTS:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.26	0.36	¯0.49	¯0.59	¯0.17	¯1.20
GAMMA DIST	0.30	0.25	¯1.03	¯0.65	0.15	¯0.31
F A S T	0.67	0.28	1.58	0.28	¯1.07	¯1.09

ACTUAL ST. DEV. - ST. ERROR OF ESTIMATE OF LOS DIST.:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	¯0.05	¯0.11	0.08	0.25	¯0.01	¯0.61
GAMMA DIST	¯0.06	¯0.17	0.00	0.19	¯0.68	¯0.70
F A S T	¯0.10	¯0.03	¯0.44	¯0.34	¯0.15	¯0.42

K-S STATISTIC:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.16	0.17	0.11	0.08	0.12	0.27
GAMMA DIST	0.25	0.27	0.21	0.09	0.18	0.15
F A S T	0.45	0.18	0.24	0.07	0.30	0.29

RATING=0 *YEAR*=1976

ACTUAL - ESTIMATED MEAN LOS OF ADVANCEMENTS:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.38	0.62	0.30	0.16	0.12	0.00
GAMMA DIST	0.28	0.80	0.03	0.16	0.21	0.15
F A S T	0.72	0.81	1.28	1.15	¯0.21	¯0.67

ACTUAL ST. DEV. - ST. ERROR OF ESTIMATE OF LOS DIST.:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.21	0.31	0.04	¯0.18	¯0.06	¯0.41
GAMMA DIST	0.29	0.26	¯0.15	¯0.18	0.11	¯0.41
F A S T	0.19	0.08	¯0.09	¯0.81	¯0.53	¯0.77

K-S STATISTIC:

MODEL	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
REGRESSION	0.11	0.15	0.04	0.03	0.02	0.03
GAMMA DIST	0.12	0.27	0.06	0.06	0.06	0.09
F A S T	0.28	0.20	0.17	0.14	0.05	0.10

H. Mean LOS of Advancements as a Function of Volume.

From the start of this research effort an answer has been sought to the question: "How does the LOS distribution of advancements depend on the volume of advancements in the Enlisted Force?" As the previous report explained (Milch, 1976) such an answer is not extractable from the data directly, because the dependence on volume is confounded by dependence on other variables, such as inventories. The construction of a model, however, makes it possible to hold other variables fixed, while the LOS distribution of advancements is observed as a function of volume alone. This was achieved with the regression model and reported previously (Milch, 1976). Here, the regression model and the newly constructed model using the gamma distribution are compared in their ability to display this dependence.

In Appendix F the mean LOS values are graphed as a function of volume of advancements as provided by the regression model and the gamma distribution model. Both models use the net inventory distribution for FY 1976. The actual mean LOS of advancees in FY 1976 is also plotted as the single data point relevant to the curves shown. In addition, the mean LOS values of advancements as provided by the FAST model is also shown as a horizontal straight line. The range of volume for each of the twenty-four cases (six pay grades for each of three ratings and ALLNAVY) is approximately the range of volume that occurred historically during FY 1966-76. These curves show that the gamma distribution model may be used equally well as the regression model to display the dependence of mean LOS of advancements on volume.

To examine this dependence closer another set of graphs is shown in Appendix G. These twenty-four graphs display a sample of the various types of curves that result when different net inventory distributions are used to show the dependence of mean LOS of advancements on volume through the gamma distribution model. Each graph shows three curves which are based on the net inventory distributions of FY 1976 and two other FY's. These latter FY's are selected for each of the twenty-four cases in such a way that they produce the two most extreme curves among the eleven curves that are based on the net inventory distributions of FY's 1966-76.

In most large volume pay grades, such as pay grades E4 and E5 of rating 300, pay grades E4, E5, E6 of rating 1500, and pay grade E4 of rating 1800, all curves are of the decreasing type. This type of curves was originally anticipated according to the rationale that large volume of advancements forces the system to promote younger personnel. In the remaining cases of the three ratings, however, this rule does not always apply. For example, in the case of pay grade E7 of rating 300, both decreasing and increasing type of curves appear. As the information provided below each graph testifies, for small volumes the mean LOS of advancements is reasonably close to the mean LOS of net inventory: low (high) mean LOS of net inventory implies low (high) mean LOS of advancements. As the volume increases, however, the mean LOS of advancement will increase or decrease depending on whether the mean LOS was low or high for small volume. For example, for pay grade E7 of rating 300, the FY's 1966 and 1974 both had relatively small volume of advancements: 39 and 150. These years had widely differing mean LOS of net inventory values: 10.56 and 13.07 years. The corresponding mean LOS values of advancements display the same discrepancy: 11.05 and 14.41 years. The two curves have correspondingly differing behavior: the curve using the FY 1966 net inventory LOS distribution is increasing, while that of FY 1974 is decreasing with volume.

III. Conclusions.

The final conclusion of the analysis of Section II. is that the gamma distribution model may be used to predict the number of advancements by LOS in place of the regression model without significant loss of accuracy in most cases. Both models have the advantage over currently used methodology of being sensitive to changes in the volume of advancements. Both models may also be used to explain the two types of functions (decreasing and increasing) that occur when plotting mean LOS values of advancements vs. volume of advancements. The main advantage of the gamma distribution model over the regression model is the use of only fifteen parameters vs. ninety-three for the regression model for each pay grade of each rating. For these reasons, current plans include extending this analysis to all ratings of the Navy Enlisted Force and adapting FAST to use the gamma distribution model to compute advancements by LOS. Whether or not this will be accomplished depends on preliminary tests to establish the supposed superiority of this model over present methodology.

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Appendix A. Proofs of Statements 2 and 3.

Statement 2.

$$E(F_i | V) = f_i(V) \tag{12}$$

$$\mathrm{Var}(F_i | V) = \frac{1}{V} f_i(V) \; [1 - f_i(V)] \tag{13}$$

Proof:

From the definition of F_i and Formula (6)

$$E(F_i | V) = \frac{1}{V} E(A_i \;\; V) = \frac{1}{V} \sum_{j=1}^{V} E(U_{ij} | V)$$

Then using Formula (10) and the fact that U_{ij}, $1 \leq j \leq V$, are i.i.d. when V is given, result (12) follows. The same facts and formulas and Formula (11) are used to show that

$$\mathrm{Var}(F_i | V) = \frac{1}{V^2} \mathrm{Var}(A_i | V) = \frac{1}{V^2} \sum_{j=1}^{V} \mathrm{Var}(U_{ij} | V)$$

$$= \frac{1}{V^2} V f_i(V) \; [1 - f_i(V)]$$

This proves Formula (13).

Statement 3.

For $1 \leq j \leq V$ and any r

$$K_r = \sum_{i=1}^{31} i^r F_i = \frac{1}{V} \sum_{V=1}^{V} L_j^r \; . \tag{15}$$

Proof:

$$K_r = \sum_{i=1}^{31} i^r F_i = \frac{1}{V} \sum_{i=1}^{31} i^r A_i = \frac{1}{V} \sum_{i=1}^{31} i^r \sum_{j=1}^{V} U_{ij}$$

$$= \frac{1}{V} \sum_{j=1}^{V} \sum_{i=1}^{31} i^r U_{ij} = \frac{1}{V} \sum_{j=1}^{V} L^r{}_j$$

having used Formulas (6) and (9).

Appendix B. A Technical Clarification

The definition given for the distribution of the joint random variables $(\mathcal{B}_r, \mathcal{I}_r)$ in Section II.E. (Formula (27)) is technically imprecise. This is so, because in the definition of the probability mass function the values assigned to the random variable must be distinct. Neither the β_i nor the I_i, $1 \leq i \leq 31$, in Formula (27) are necessarily distinct. In fact, many of β_i as well as the I_i values are usually zeroes. This technical difficulty may be eliminated only at the expense of some additional notation.

Let $\beta_1', \ldots, \beta_s'$ and $I_i', \ldots, I_t'$ be all the distinct values among $\beta_1, \ldots, \beta_{31}$ and $I_1, \ldots, I_{31}$, respectively. Clearly $1 \leq s$, $t \leq 31$. For every pair of indices (i, j) the set of indices k for which both $\beta_k = \beta_i'$ and $I_k = I_j'$ is introduced as

$$C_{ij} = \{k: \ \beta_k = \beta_i' \text{ and } I_k = I_j', \ 1 \leq k \leq 31\}$$

where $i = 1, \ldots, s$ and $j = 1, \ldots, t$

Now, definition (27) may be corrected:

$$P\,(\mathcal{B}_r = \beta_i', \mathcal{I}_r = I_j') = \frac{1}{n_r} \sum_{k \in C_{ij}} k^r$$

for $i = 1, \ldots, s$ and $j = 1, \ldots, t$ and $r = 0, 1, 2$. In order to compute the marginal distributions further notation is required. For $i = 1, \ldots, s$

$$D_i = \{k: \ \beta_k = \beta_i', \ 1 \leq k \leq 31\}$$

and for $j = 1, \ldots, t$

$$E_j = \{k: \ I_k = I_j', \ 1 \leq k \leq 31\}$$

Then, for $i = 1,\ldots,s$,

$$P(\mathcal{B}_r = \beta_i') = \sum_{j=1}^{t} P(\mathcal{B}_r = \beta_i', \mathcal{I}_r = I_j')$$

$$= \sum_{j=1}^{t} \frac{1}{n_r} \sum_{k \varepsilon C_{ij}} k^r = \frac{1}{n_r} \sum_{k \varepsilon D_i} k^r \quad ,$$

since the union of all C_{ij}, $1 \leq j \leq t$ is D_i:

$$D_i = \bigcup_{j=1}^{t} C_{ij} \quad .$$

For similar reasons, for $j = 1,\ldots, t$,

$$P(\mathcal{I}_r = I_j') = \frac{1}{n_r} \sum_{k \varepsilon E_j} k^r \quad .$$

Formulas ( ) and ( ) are the precise versions of Formula (28) of Section II.E. The fact that all the formulas involving the moments of $(\mathcal{B}_r, \mathcal{I}_r)$ are correct as given in Section II.E. may be seen without difficulty. For example,

$$E(\mathcal{B}_r^m \mathcal{I}_r^n) = \sum_{i=1}^{s} \sum_{j=1}^{t} (\beta_i')^m (I_j')^n \frac{1}{n_r} \sum_{k \varepsilon C_{ij}} k^r \quad .$$

But short reflection on the definition of C_{ij} shows that

$$\frac{1}{n_r} \sum_{i=1}^{s} \sum_{j=1}^{t} \sum_{k \varepsilon C_{ij}} (\beta_i')^m (I_j')^n k^r = \frac{1}{n_r} \sum_{i=1}^{31} \beta_i^m I_i^n i^r = S_r(\beta^m I^n) \quad .$$

This verifies Formula (29), the most general moment formula in Section II.E.

APPENDIX C

THE CORRELATION COEFFICIENT: RHO0

RATING=300

YEAR	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
1966	¯0.0673	0.1013	0.0225	¯0.0448	0.3528	¯0.0025
1967	¯0.0796	0.1026	0.0990	0.0019	0.4067	0.2207
1968	¯0.0827	¯0.0251	0.0878	¯0.0015	0.3329	0.4108
1969	¯0.0949	¯0.0354	0.0035	¯0.0077	0.3366	0.2646
1970	¯0.1113	¯0.0151	0.0385	0.0058	0.3922	0.1759
1971	¯0.0911	0.0619	0.0012	0.0147	0.2856	0.3824
1972	¯0.0942	0.1078	¯0.0111	0.0666	0.3261	0.3558
1973	¯0.0955	0.1418	0.0409	0.0683	0.2921	0.1807
1974	¯0.0638	0.0865	0.0478	0.0781	0.3174	0.1280
1975	¯0.1009	0.0850	0.0879	0.0355	0.2982	0.1365
1976	¯0.1139	0.0521	0.0931	0.0216	0.3037	¯0.1307

RATING=1500

1966	0.1155	0.2372	0.0594	¯0.0123	0.3081	0.3988
1967	0.1031	0.2294	0.0295	0.0170	0.3000	0.3591
1968	0.1267	0.1294	0.0024	0.0247	0.3547	0.4821
1969	0.0947	0.1575	¯0.0086	¯0.0042	0.3181	0.5809
1970	0.0650	0.1414	¯0.0349	0.0264	0.3005	0.5013
1971	0.0775	0.1901	¯0.0030	0.1108	0.3506	0.5378
1972	0.1176	0.2336	¯0.0082	0.1356	0.3624	0.5860
1973	0.1206	0.2732	0.0241	0.1708	0.3680	0.4953
1974	0.1424	0.2154	¯0.0332	0.2139	0.3754	0.5921
1975	0.0927	0.2519	0.0165	0.2292	0.3836	0.5322
1976	0.1091	0.1985	0.0649	0.2298	0.3332	0.4587

RATING=1800

1966	¯0.0241	0.1807	0.1341	0.1551	0.2247	¯0.0165
1967	¯0.0271	0.2130	0.1658	0.2291	0.0986	¯0.0149
1968	¯0.0073	¯0.0132	0.2111	0.3169	0.2244	0.1495
1969	¯0.0219	¯0.0827	0.0278	0.2142	0.1843	¯0.0445
1970	¯0.0335	¯0.0091	0.1054	0.2670	0.0787	0.1202
1971	¯0.0292	¯0.0254	0.1204	0.3106	0.1194	0.0842
1972	¯0.0334	0.0184	¯0.0243	0.3270	0.2157	0.0149
1973	¯0.0347	0.0331	0.0744	0.3243	0.2058	0.0246
1974	¯0.0282	¯0.0309	¯0.0042	0.3082	0.2186	0.0383
1975	¯0.0282	0.0121	¯0.0005	0.3808	0.2148	¯0.0517
1976	¯0.0280	0.0460	0.0290	0.3792	0.1570	0.0582

RATING=0

1966	0.2635	0.3760	0.1919	0.3725	0.4532	0.1763
1967	0.2974	0.3697	0.1238	0.3942	0.4828	0.2285
1968	0.2795	0.1485	0.1816	0.4355	0.5342	0.2433
1969	0.2996	0.1515	0.0213	0.3942	0.4371	0.2317
1970	0.3066	0.1716	0.0929	0.4076	0.4024	0.2306
1971	0.2887	0.2080	0.0518	0.4786	0.4187	0.2224
1972	0.2476	0.3324	0.0938	0.5448	0.4170	0.2162
1973	0.2641	0.3586	0.0907	0.5510	0.4156	0.1964
1974	0.2877	0.2458	0.0538	0.5925	0.4173	0.2045
1975	0.2698	0.2774	0.0863	0.5962	0.4419	0.1938
1976	0.2850	0.2469	0.0823	0.5620	0.4598	0.1894

THE CORRELATION COEFFICIENT: RHO1

YEAR	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
			RATING=300			
1966	0.0266	0.0987	0.0014	¯0.0732	0.2897	¯0.0587
1967	0.0242	0.0898	0.0895	¯0.0533	0.3413	0.1333
1968	0.0180	0.0332	0.1337	¯0.0518	0.2810	0.3480
1969	0.0106	0.0225	0.0750	¯0.0565	0.2580	0.2122
1970	0.0013	0.0302	0.1118	¯0.0510	0.3103	0.0703
1971	0.0102	0.0526	0.0751	¯0.0488	0.2014	0.2932
1972	0.0124	0.0702	0.0475	¯0.0262	0.2505	0.3000
1973	0.0152	0.0751	0.0941	¯0.0270	0.2141	0.0870
1974	0.0298	0.0645	0.0759	¯0.0179	0.2505	0.0499
1975	0.0138	0.0643	0.1103	¯0.0431	0.2244	0.0938
1976	0.0147	0.0686	0.0764	¯0.0487	0.2483	¯0.1825
			RATING=1500			
1966	0.0986	0.1669	0.0383	0.1516	0.4253	0.3212
1967	0.0894	0.1673	0.0198	0.1716	0.4196	0.2703
1968	0.0980	0.1173	0.0217	0.1994	0.4897	0.4275
1969	0.0820	0.1204	0.0405	0.1713	0.4382	0.5333
1970	0.0593	0.1137	0.0105	0.2130	0.4016	0.4484
1971	0.0652	0.1271	0.0259	0.3000	0.4552	0.4855
1972	0.0904	0.1393	0.0298	0.3269	0.4618	0.5262
1973	0.0996	0.1460	0.0298	0.3537	0.4739	0.4408
1974	0.1070	0.1401	0.0072	0.3965	0.4801	0.5475
1975	0.0842	0.1542	0.0304	0.4012	0.4871	0.4837
1976	0.0988	0.1703	0.0502	0.3951	0.4350	0.3927
			RATING=1800			
1966	0.0524	0.1966	0.1600	0.1620	0.4590	¯0.1300
1967	0.0502	0.2206	0.1932	0.1954	0.2889	¯0.1307
1968	0.0605	0.1330	0.2551	0.2784	0.4569	0.0561
1969	0.0395	0.0457	0.1814	0.2066	0.4028	¯0.1769
1970	0.0226	0.0919	0.2004	0.2460	0.2385	0.0298
1971	0.0253	0.0511	0.2647	0.2855	0.2902	¯0.0514
1972	0.0211	0.0596	0.1455	0.2944	0.4124	¯0.1062
1973	0.0191	0.0728	0.1943	0.2975	0.3866	¯0.0900
1974	0.0296	0.0505	0.1440	0.2825	0.4121	¯0.0744
1975	0.0281	0.0645	0.1352	0.3429	0.4232	¯0.1692
1976	0.0304	0.0676	0.1136	0.3399	0.3709	¯0.0327
			RATING=0			
1966	0.0595	0.3982	0.2112	0.2902	0.3790	0.2529
1967	0.0741	0.3880	0.1762	0.3081	0.3918	0.3602
1968	0.0732	0.2644	0.2349	0.3669	0.4646	0.3876
1969	0.0772	0.1994	0.1578	0.3473	0.3564	0.3629
1970	0.0776	0.2040	0.1647	0.3503	0.3090	0.3592
1971	0.0724	0.2144	0.1633	0.4112	0.3242	0.3295
1972	0.0489	0.2664	0.1927	0.4663	0.3166	0.3117
1973	0.0558	0.2945	0.1774	0.4647	0.3228	0.2785
1974	0.0707	0.2624	0.1475	0.4875	0.3227	0.2952
1975	0.0667	0.2730	0.1583	0.4953	0.3484	0.2756
1976	0.0706	0.2801	0.1417	0.4692	0.3731	0.2705

THE CORRELATION COEFFICIENT: RHO2

RATING=300

YEAR	*E*4	*E*5	*E*6	*E*7	*E*8	*E*9
1966	0.0318	0.0845	0.0096	¯0.0689	0.2854	¯0.0415
1967	0.0342	0.0710	0.1122	¯0.0603	0.3335	0.1120
1968	0.0240	0.0341	0.1845	¯0.0558	0.2897	0.3302
1969	0.0219	0.0260	0.1451	¯0.0594	0.2372	0.2157
1970	0.0165	0.0265	0.1901	¯0.0570	0.2850	0.0254
1971	0.0184	0.0297	0.1583	¯0.0580	0.1787	0.2402
1972	0.0204	0.0453	0.1087	¯0.0506	0.2313	0.2854
1973	0.0261	0.0408	0.1579	¯0.0523	0.1961	0.0511
1974	0.0328	0.0399	0.1102	¯0.0424	0.2413	0.0355
1975	0.0280	0.0408	0.1494	¯0.0604	0.2096	_0.1103
1976	0.0430	0.0506	0.0884	¯0.0630	0.2471	¯0.1544

RATING=1500

YEAR	*E*4	*E*5	*E*6	*E*7	*E*8	*E*9
1966	0.0634	0.1197	0.0361	0.2379	0.3908	0.2580
1967	0.0562	0.1199	0.0281	0.2432	0.3819	0.1974
1968	0.0588	0.0885	0.0366	0.2750	0.4575	0.3789
1969	0.0517	0.0807	0.0763	0.2479	0.4081	0.4844
1970	0.0358	0.0752	0.0365	0.2825	0.3664	0.4015
1971	0.0353	0.0735	0.0455	0.3469	0.4158	0.4347
1972	0.0485	0.0782	0.0548	0.3653	0.4187	0.4567
1973	0.0569	0.0777	0.0416	0.3793	0.4364	0.3890
1974	0.0626	0.0824	0.0383	0.4112	0.4411	0.4983
1975	0.0541	0.0951	0.0494	0.4068	0.4478	0.4359
1976	0.0695	0.1283	0.0553	0.3993	0.4054	0.3289

RATING=1800

YEAR	*E*4	*E*5	*E*6	*E*7	*E*8	*E*9
1966	0.0744	0.1895	0.1840	0.2201	0.5092	¯0.1402
1967	0.0661	0.2106	0.2205	0.2126	0.3390	¯0.1382
1968	0.0847	0.1941	0.2906	0.2798	0.5008	_0.0531
1969	0.0473	0.0954	0.2567	0.2400	0.4434	¯0.1927
1970	0.0282	0.1417	0.2523	0.2655	0.2804	_0.0337
1971	0.0238	0.0703	0.3353	0.2975	0.3289	¯0.0697
1972	0.0259	0.0657	0.2483	0.2985	0.4471	¯0.1053
1973	0.0206	0.0764	0.2675	0.3069	0.4111	¯0.1080
1974	0.0346	0.0636	0.2346	0.2917	0.4407	¯0.0789
1975	0.0330	0.0740	0.2179	0.3349	0.4738	¯0.1739
1976	0.0331	0.0648	0.1716	0.3315	0.4173	¯0.0253

RATING=0

YEAR	*E*4	*E*5	*E*6	*E*7	*E*8	*E*9
1966	0.0133	0.4182	0.2212	0.2706	0.3695	0.2618
1967	0.0176	0.3991	0.1971	0.2849	0.3742	0.4078
1968	0.0202	0.3316	0.2497	0.3444	0.4565	0.4418
1969	0.0185	0.2087	0.2210	0.3386	0.3532	0.3985
1970	0.0165	0.1970	0.2017	0.3349	0.2967	0.3979
1971	0.0163	0.1934	0.2248	0.3820	0.3032	0.3321
1972	0.0066	0.2205	0.2492	0.4234	0.2831	0.3080
1973	0.0093	0.2631	0.2293	0.4171	0.2946	0.2812
1974	0.0172	0.2534	0.2003	0.4264	0.2917	0.3013
1975	0.0194	0.2545	0.1955	0.4340	0.3155	0.2794
1976	0.0199	0.2650	0.1711	0.4167	0.3449	0.2745

AVERAGE RHO VALUES OVER THE FY'S 1972-76

RHO0

RATING	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
300	¯0.0936	0.0947	0.0517	0.0540	0.3075	0.1340
1500	0.1165	0.2345	0.0128	0.1958	0.3645	0.5329
1800	¯0.0305	0.0157	0.0149	0.3439	0.2024	0.0169
0	0.2709	0.2922	0.0814	0.5693	0.4303	0.2001

RHO1

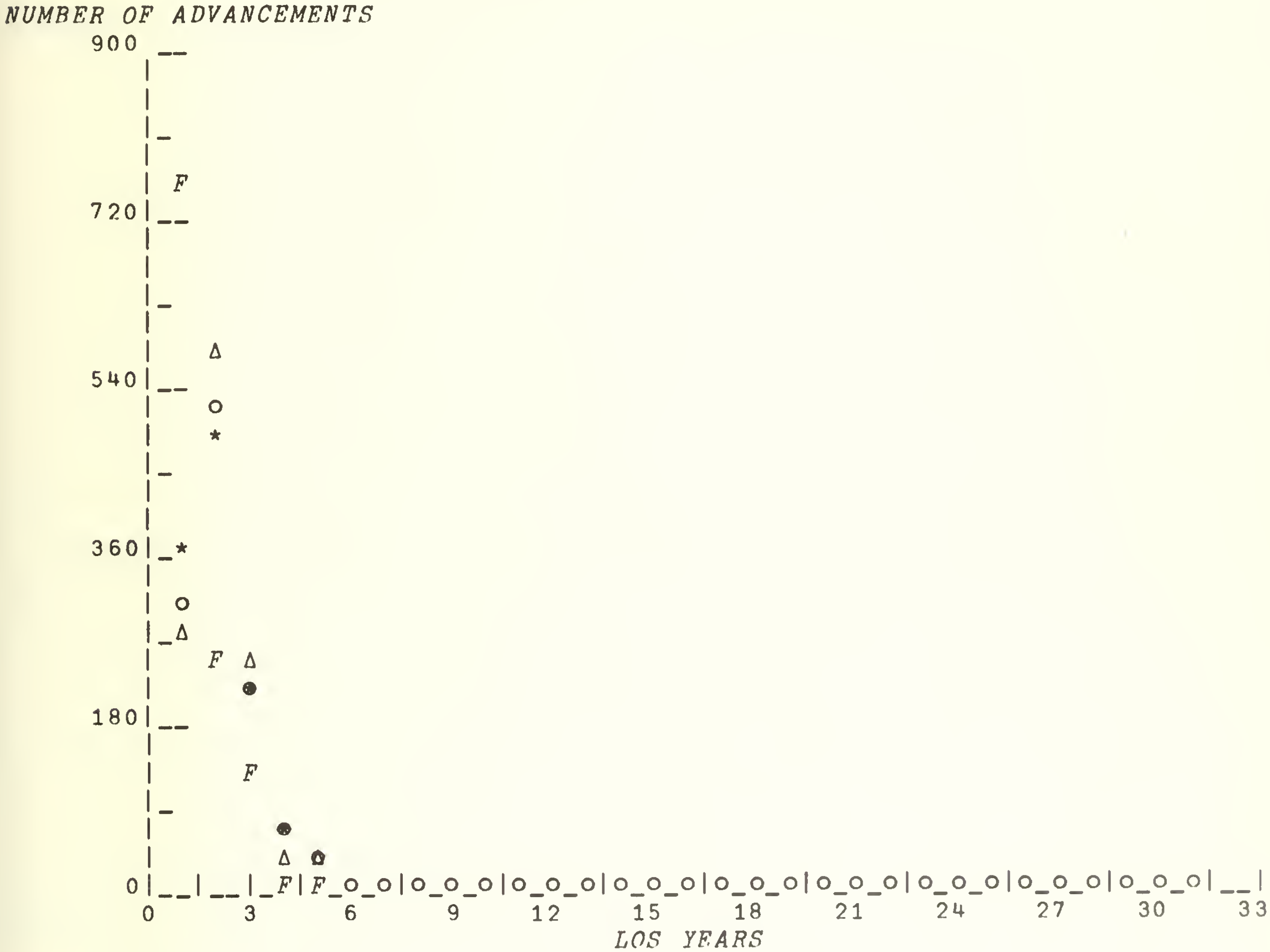
RATING	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
300	0.0172	0.0686	0.0808	¯0.0326	0.2375	0.0696
1500	0.0960	0.1500	0.0295	0.3747	0.4676	0.4782
1800	0.0257	0.0630	0.1465	0.3115	0.4010	¯0.0945
0	0.0626	0.2753	0.1635	0.4766	0.3367	0.2863

RHO2

RATING	*E4*	*E5*	*E6*	*E7*	*E8*	*E9*
300	0.0301	0.0435	0.1229	¯0.0537	0.2251	0.0656
1500	0.0583	0.0923	0.0479	0.3924	0.4299	0.4218
1800	0.0294	0.0689	0.2280	0.3127	0.4380	¯0.0983
0	0.0145	0.2513	0.2091	0.4235	0.3060	0.2889

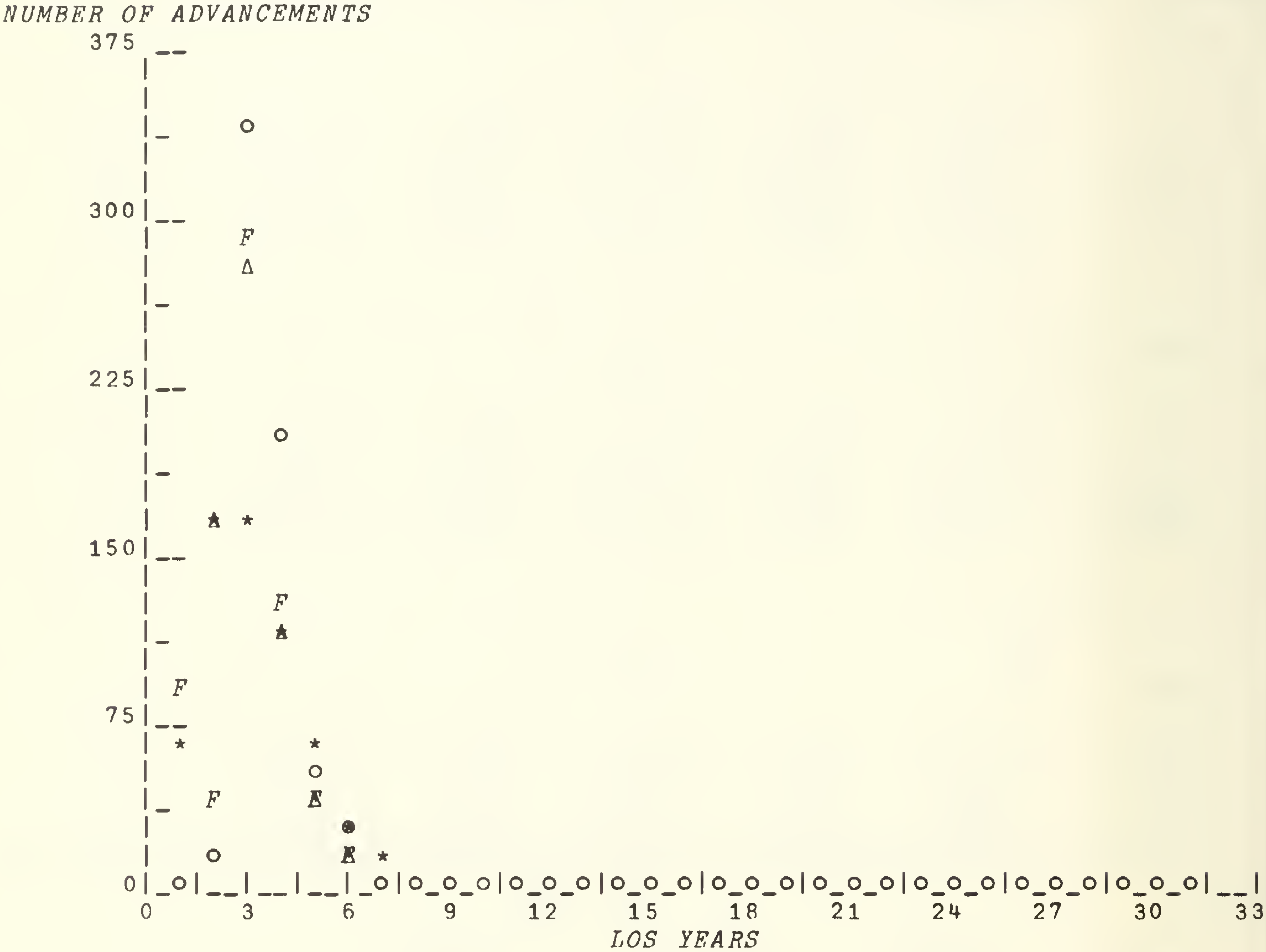
APPENDIX D

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=300 *PAY GRADE*=*E*4 *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* *:*GAMMA DIST*.(4.46 2.13) *F*: *F A S T*



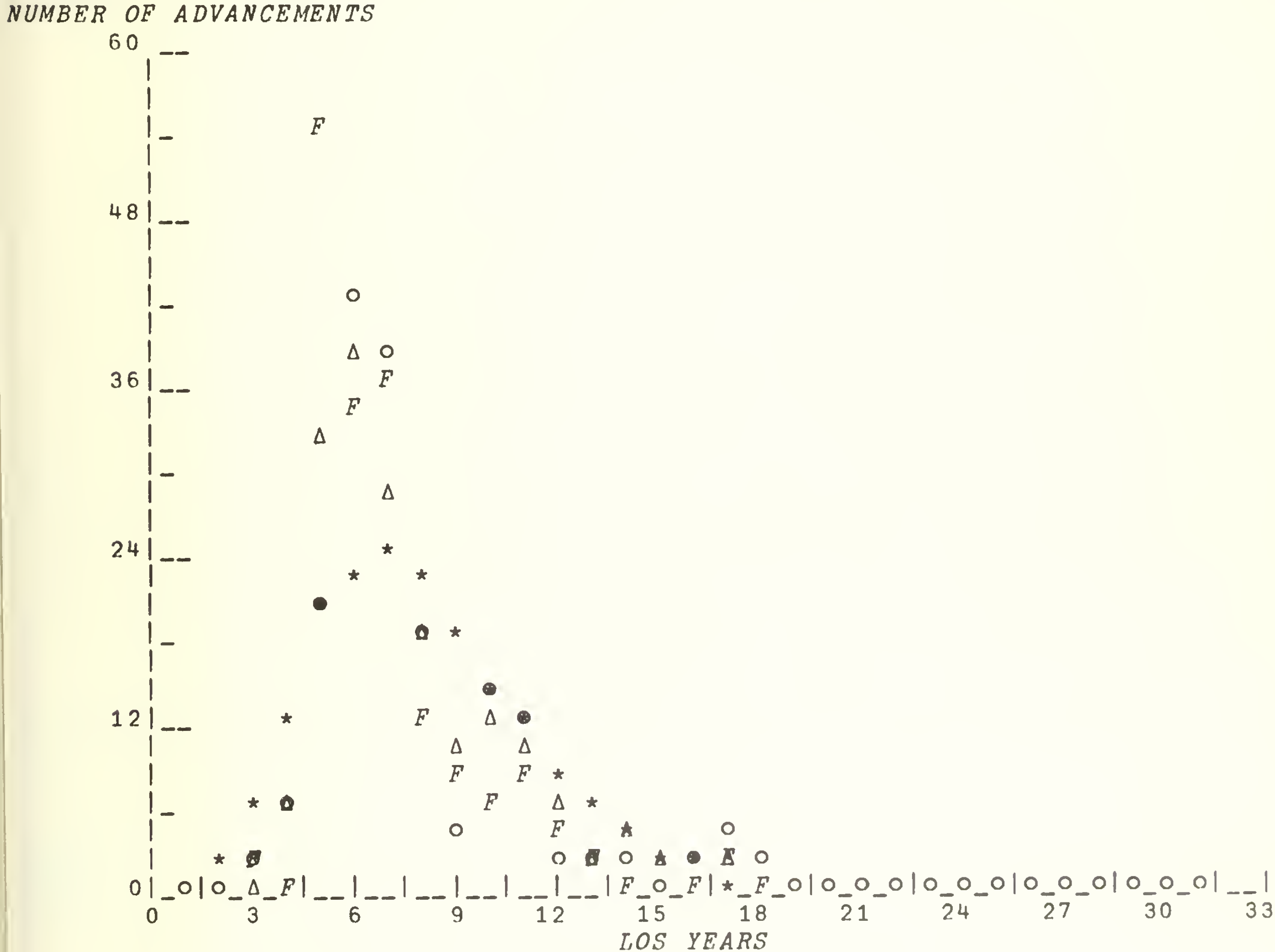
MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	1204	2.24	1.42
REGRESSION	1203	2.22	1.24
GAMMA DIST	1204	2.08	1.00
F A S T	1203	1.60	1.30

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=300 *PAY GRADE*=*E*5 *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* ⋆:*GAMMA DIST*.(4.05 1.25) *F*: *F A S T*



MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	652	3.65	1.11
REGRESSION	650	3.42	1.68
GAMMA DIST	652	3.23	1.65
F A S T	654	3.43	2.04

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=300 *PAY GRADE*=E6 *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* *:*GAMMA DIST*.(6.06 0.75) *F*: *F A S T*



MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	182	7.85	3.16
REGRESSION	180	7.68	3.00
GAMMA DIST	182	7.99	3.17
F A S T	180	7.23	2.91

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=300 PAY GRADE=E7 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(19.13 1.37) F: F A S T

MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	113	14.44	3.02
REGRESSION	112	14.21	2.78
GAMMA DIST	112	14.03	3.22
F A S T	115	12.50	4.49

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=300 *PAY GRADE*=E8 *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* *:*GAMMA DIST.*(64.37 3.82) *F*: *F A S T*

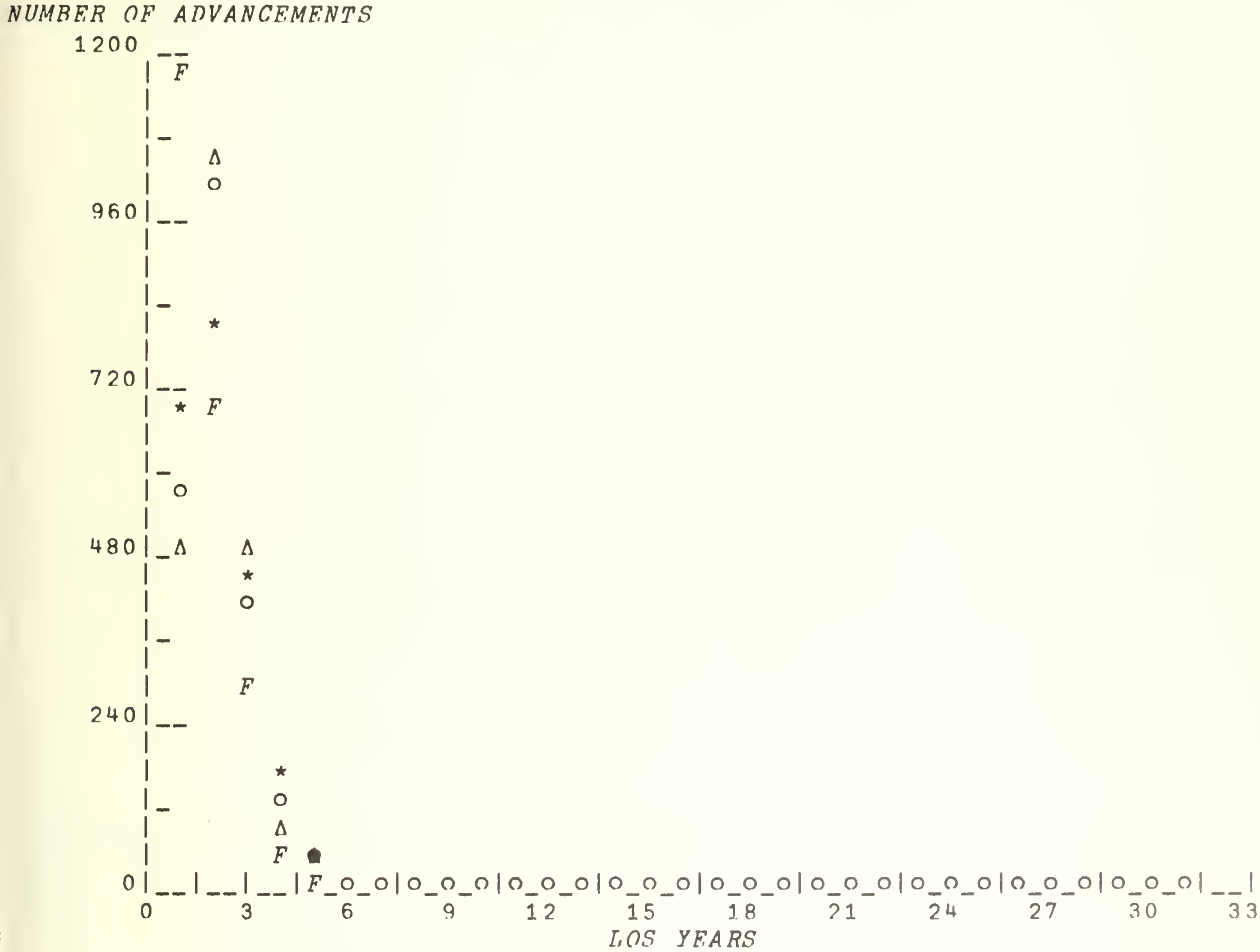
MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	19	16.37	2.29
REGRESSION	17	17.00	2.04
GAMMA DIST	20	16.75	2.14
F A S T	18	16.89	2.17

APPENDIX D (cont'd)

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=300 PAY GRADE=E9 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(98.08 5.45) F: F A S T

MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	6	18.50	0.84
REGRESSION	6	18.00	1.05
GAMMA DIST	5	18.00	1.64
F A S T	5	18.20	1.91

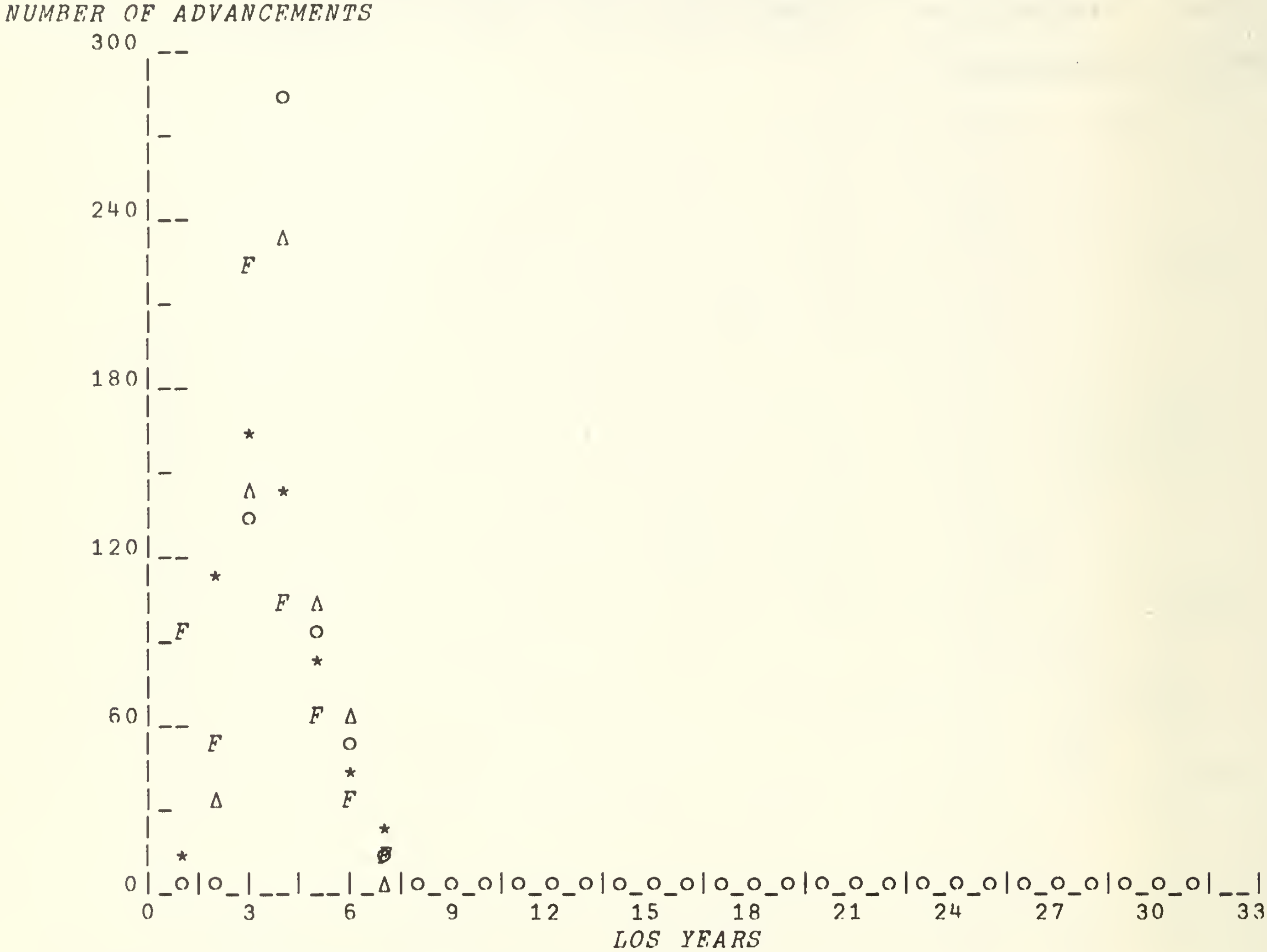
COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1500 *PAY GRADE*=*E*4 *FISCAL YEAR*=1976
o:*ACTUAL* Λ:*REGRESSION* *:*GAMMA DIST*.(3.63 1.66) *F*: *F A S T*



MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	2247	2.28	1.45
REGRESSION	2246	2.26	1.29
GAMMA DIST	2245	2.18	1.13
F A S T	2245	1.72	1.20

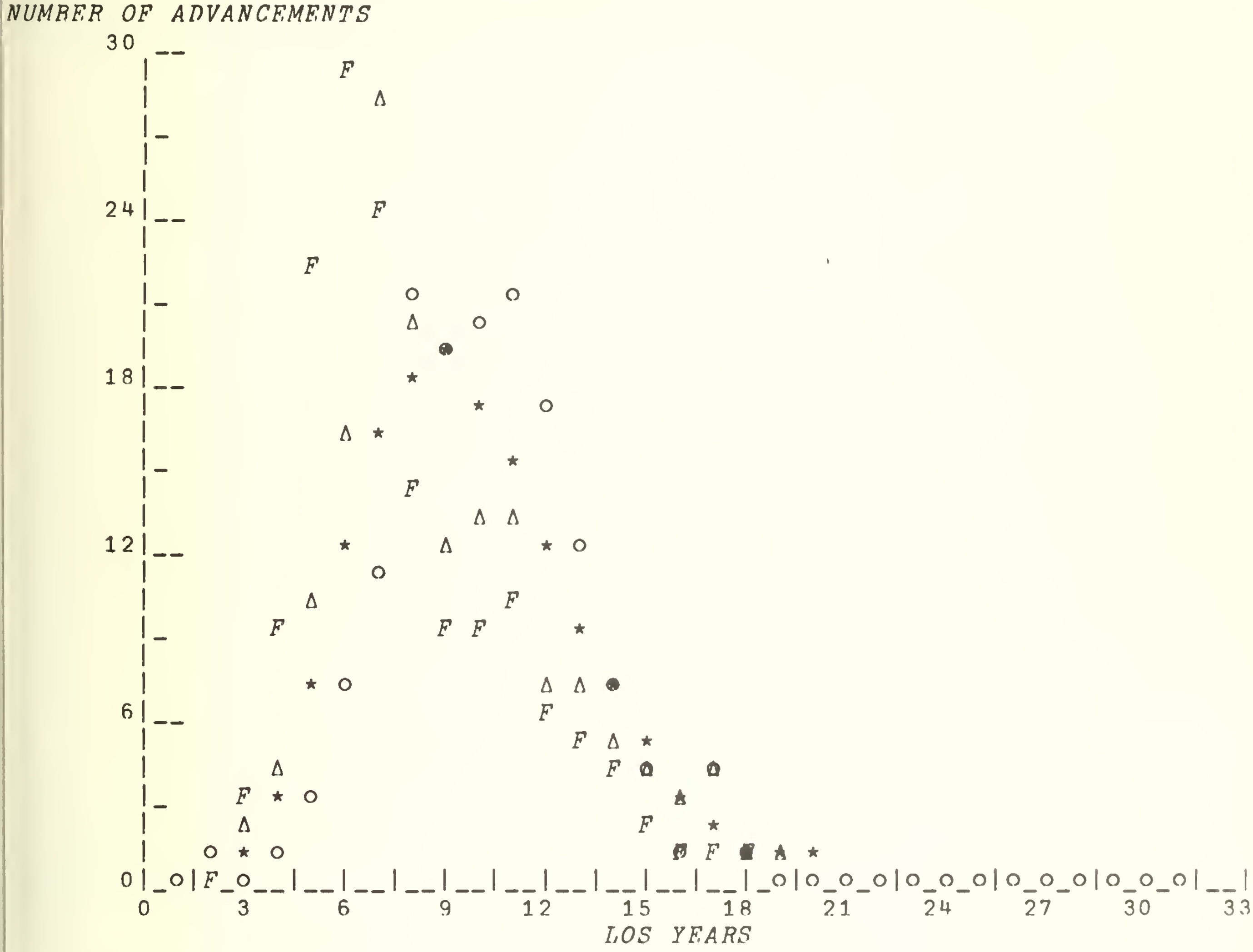
APPENDIX D (cont'd)

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1500 PAY GRADE=E5 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(5.77 1.54) F: F A S T



MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	608	4.44	1.87
REGRESSION	609	4.28	1.85
GAMMA DIST	605	3.74	1.69
F A S T	607	3.47	2.25

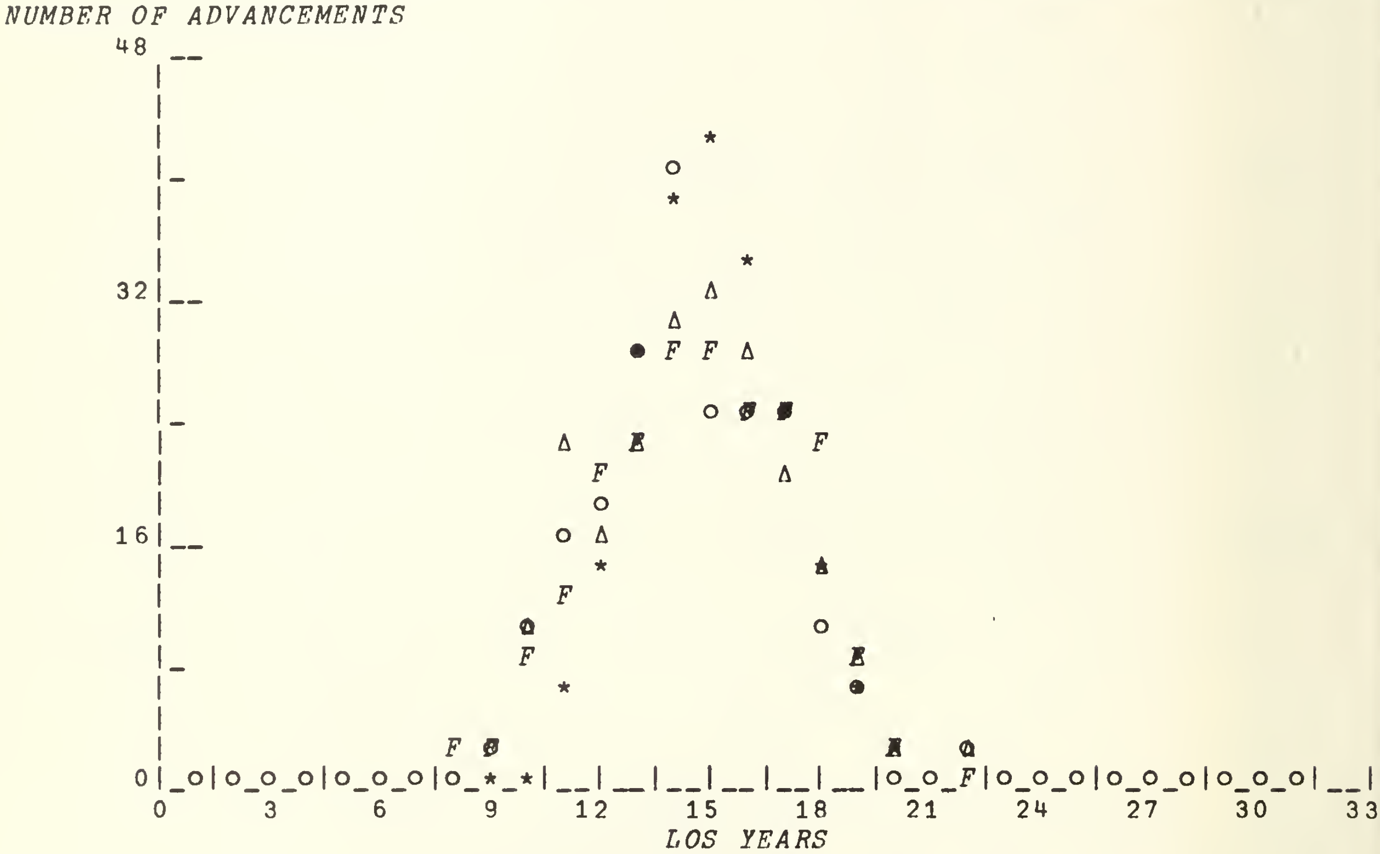
COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1500 *PAY GRADE*=*E*6 *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* ⋆:*GAMMA DIST*.(8.58 0.88) *F*: *F A S T*



MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	150	10.20	2.82
REGRESSION	150	9.09	3.56
GAMMA DIST	149	9.78	3.30
F A S T	149	7.81	3.89

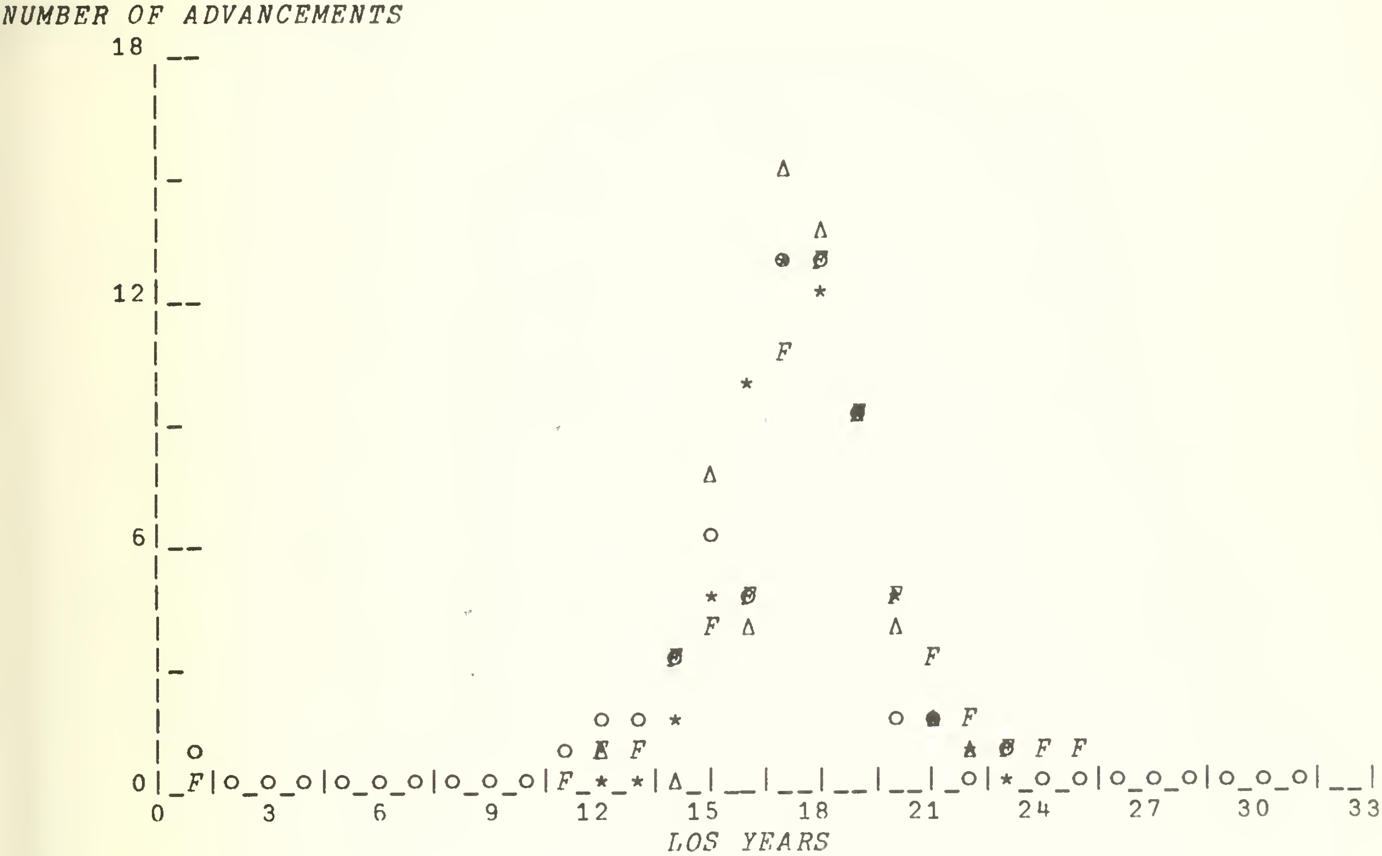
APPENDIX D (cont'd)

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1500 *PAY GRADE=E7* *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* *:*GAMMA DIST.*(52.89 3.52) *F: F A S T*



MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	216	14.36	2.65
REGRESSION	215	14.54	2.72
GAMMA DIST	215	15.02	2.14
F A S T	213	14.65	2.76

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1500 *PAY GRADE*=*E*8 *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* ⋆:*GAMMA DIST*.(87.94 5) *F*: *F A S T*

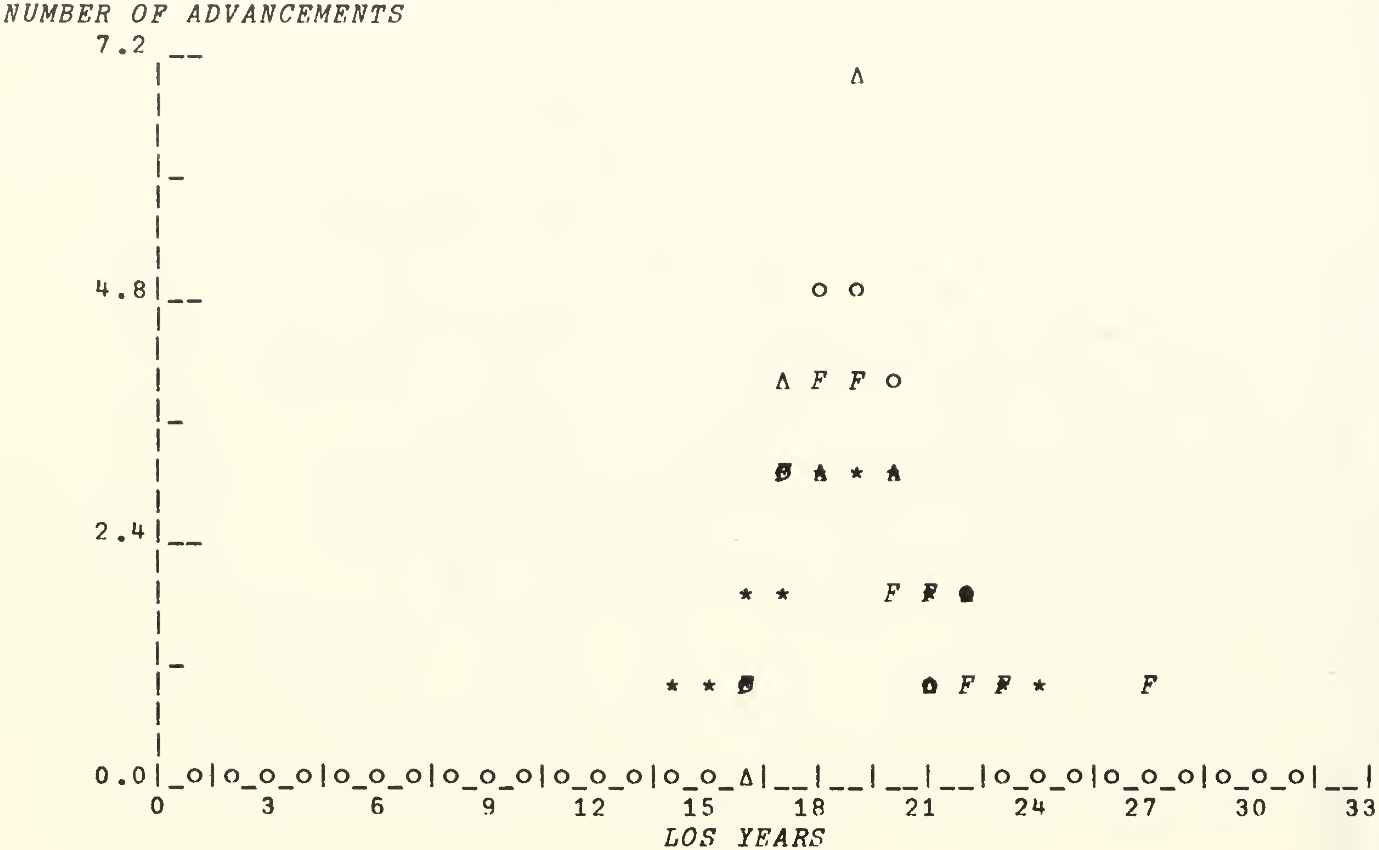


MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	60	16.75	3.08
REGRESSION	58	17.55	1.98
GAMMA DIST	59	17.54	1.94
F A S T	60	17.97	2.80

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS

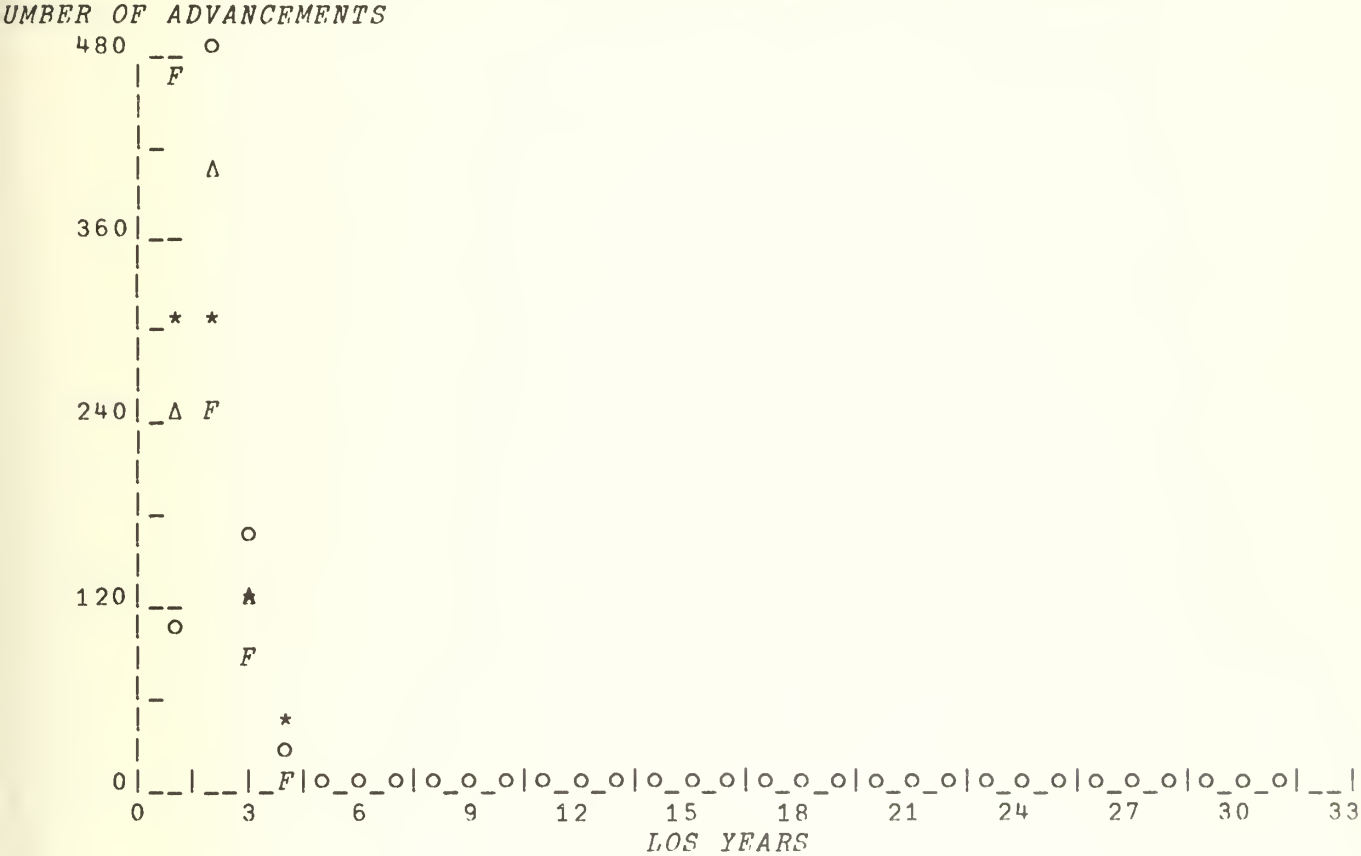
RATING=1500 *PAY GRADE*=E9 *FISCAL YEAR*=1976

o:*ACTUAL* Δ:*REGRESSION* *:*GAMMA DIST*.(44.78 2.35) *F*: *F A S T*



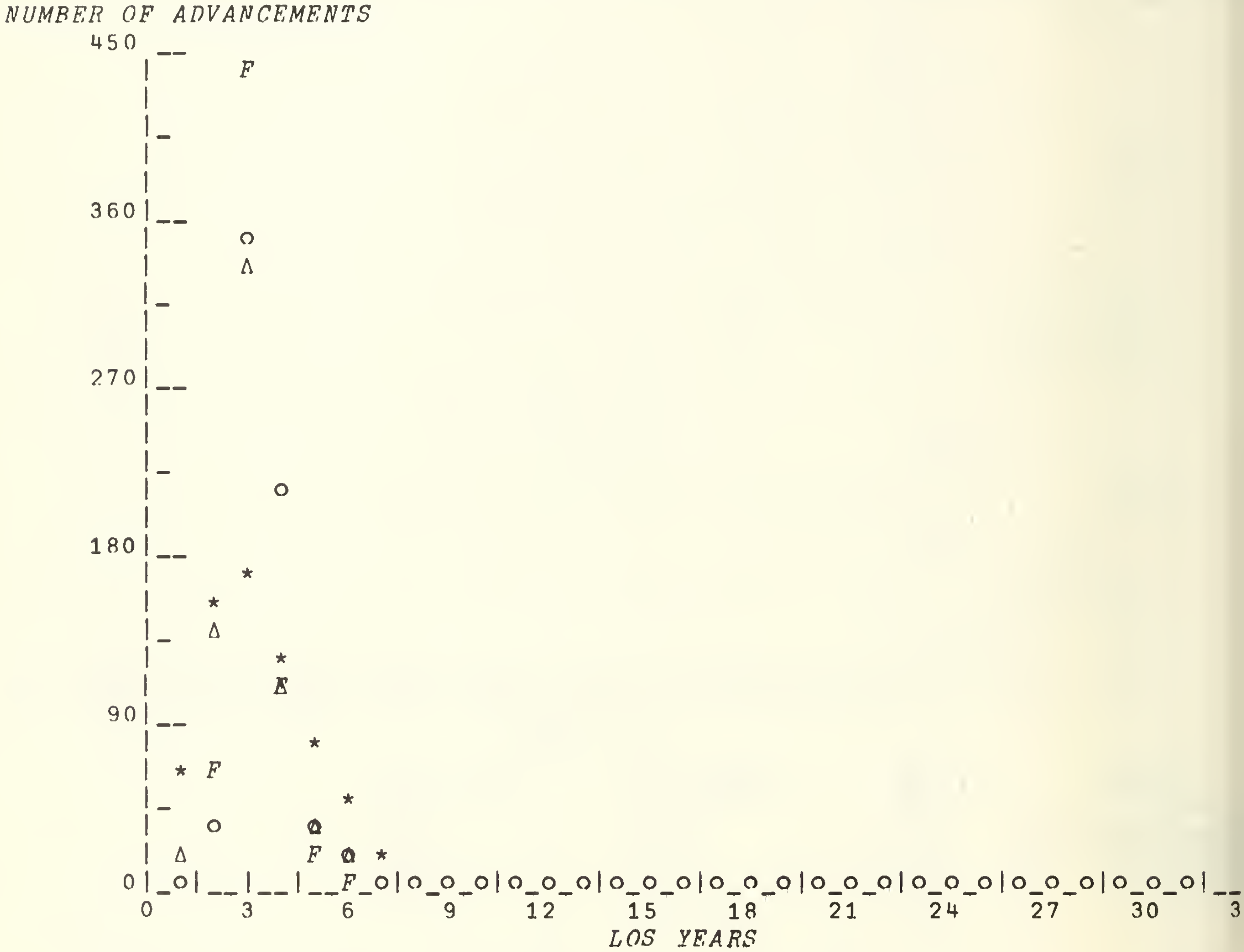
MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	21	18.90	1.61
REGRESSION	20	19.00	1.52
GAMMA DIST	21	19.00	2.65
F A S T	19	19.42	2.64

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1800 *PAY GRADE*=*E*4 *FISCAL YEAR*=1976
:*ACTUAL* Λ:*REGRESSION* *:*GAMMA DIST*.(3.68 1.9) *F*: *F A S T*

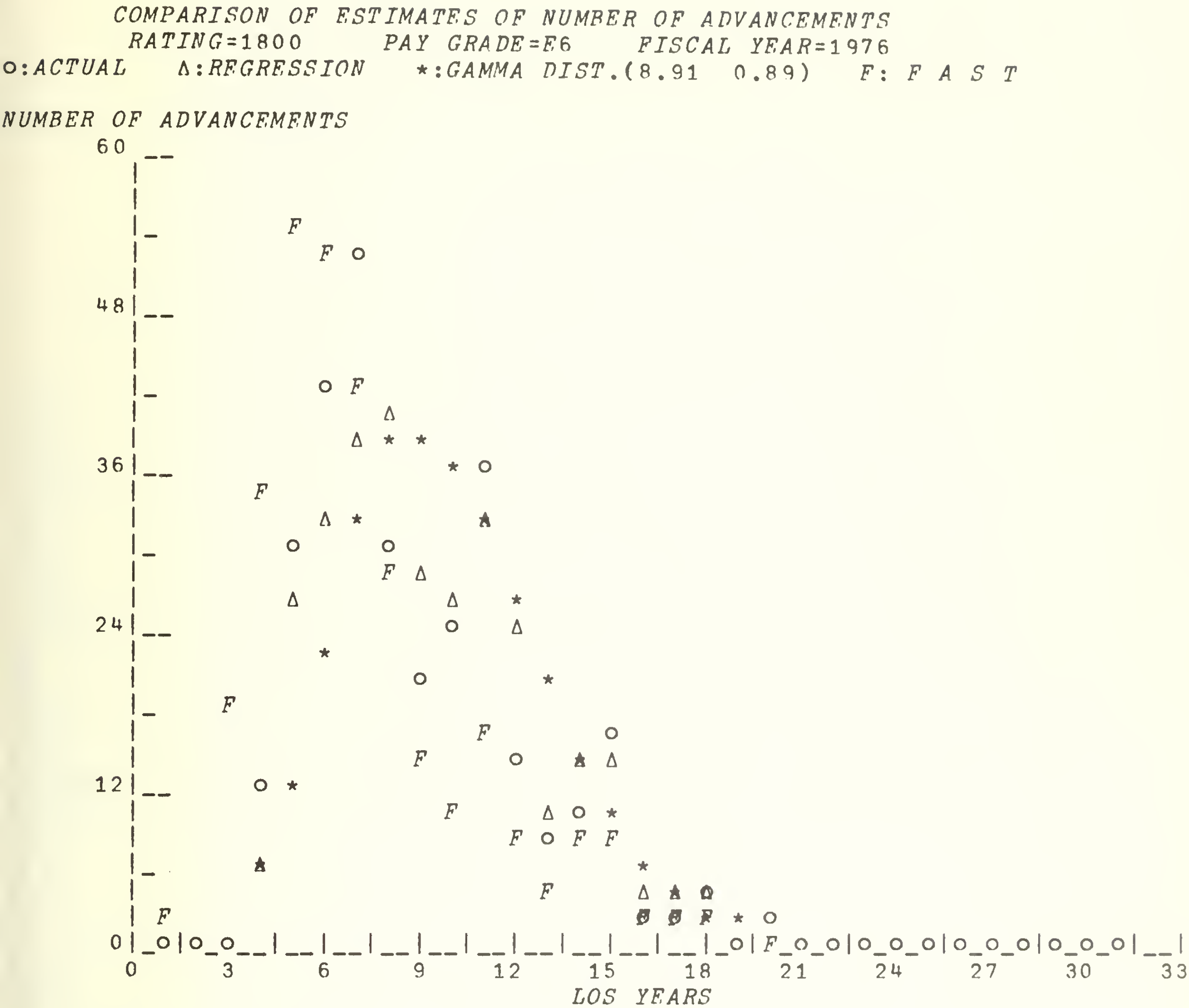


MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
CTUAL	821	2.25	0.98
EGRESSION	822	1.99	1.04
AMMA DIST	821	1.95	1.04
A S T	820	1.58	1.08

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1800 PAY GRADE=E5 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(3.65 1.04) F: F A S T

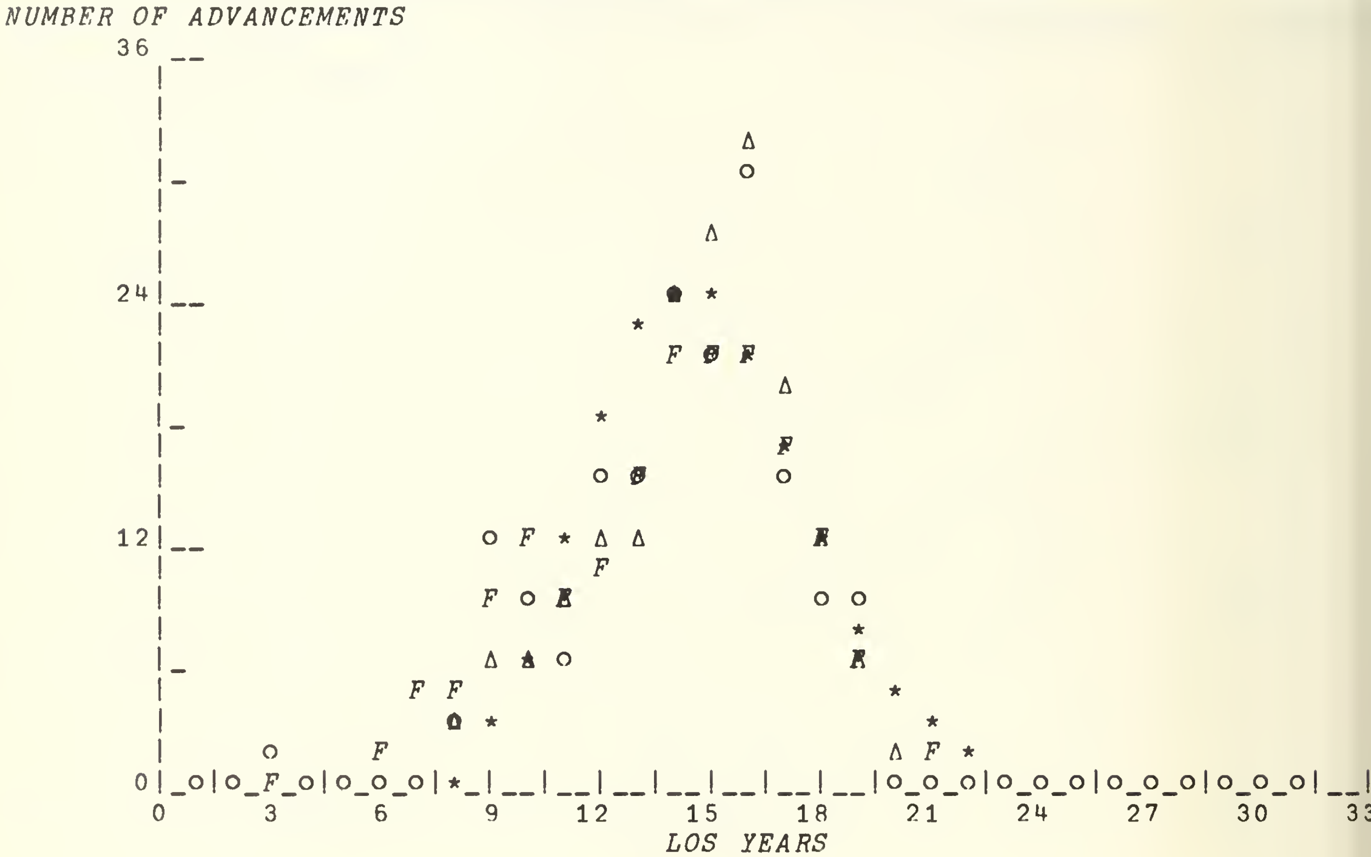


MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	704	3.77	1.68
REGRESSION	703	3.41	1.79
GAMMA DIST	703	3.51	1.85
F A S T	705	3.49	1.71



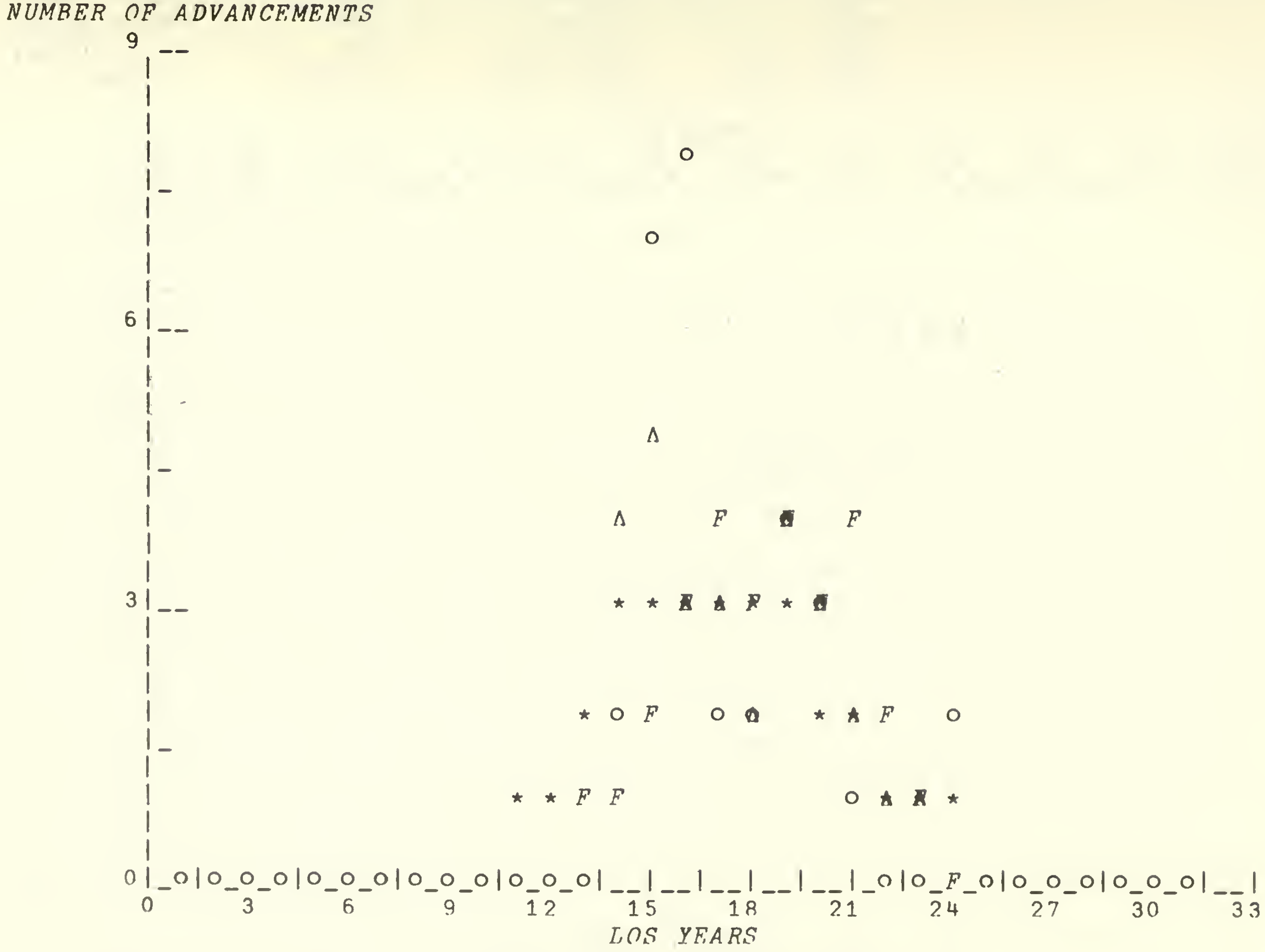
MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	314	8.93	3.43
REGRESSION	311	9.42	3.35
GAMMA DIST	312	9.96	3.44
F A S T	316	7.34	3.88

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1800 *PAY GRADE=E7* *FISCAL YEAR*=1976
o:*ACTUAL* Δ:*REGRESSION* *:*GAMMA DIST.*(25.17 1.7) *F: F A S T*



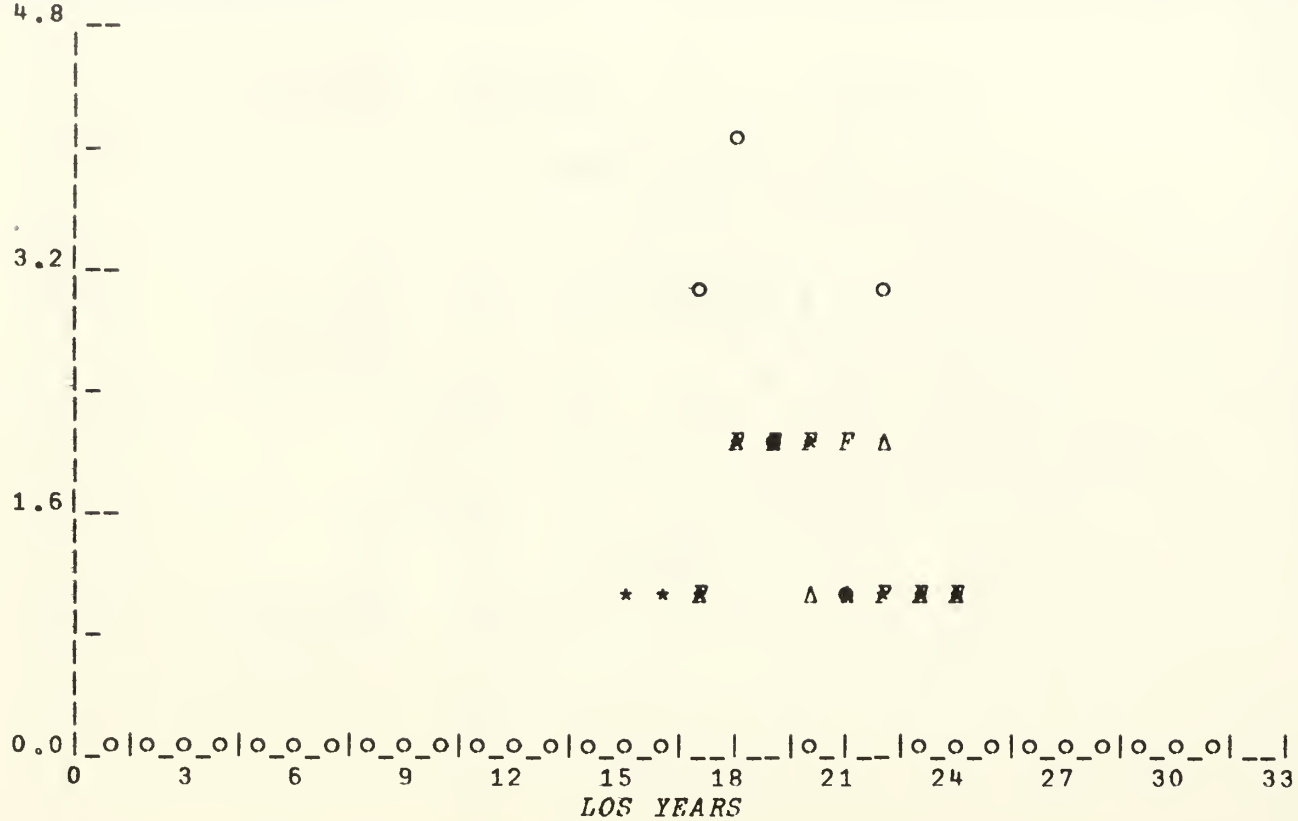
MODEL	*VOLUME*	*MEAN LOS*	*ST. ERR.*
ACTUAL	181	14.10	3.16
REGRESSION	178	14.69	2.91
GAMMA DIST	182	14.75	2.96
F A S T	179	13.82	3.50

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1800 PAY GRADE=E8 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(21.94 1.28) F: F A S T



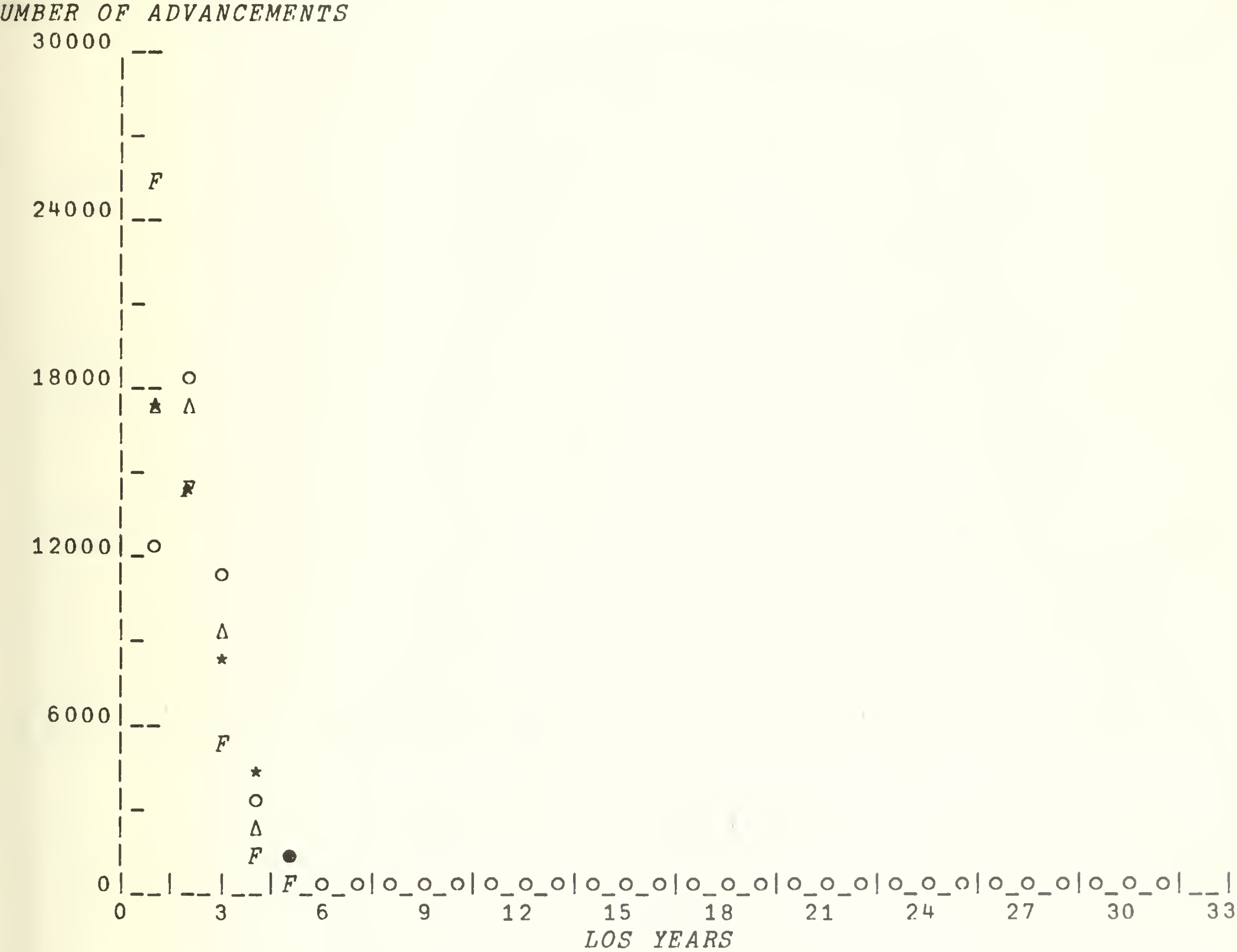
MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	31	17.29	2.65
REGRESSION	28	17.46	2.66
GAMMA DIST	29	17.14	3.32
F A S T	28	18.36	2.80

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=1800 PAY GRADE=E9 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(38.99 1.99) F: F A S T



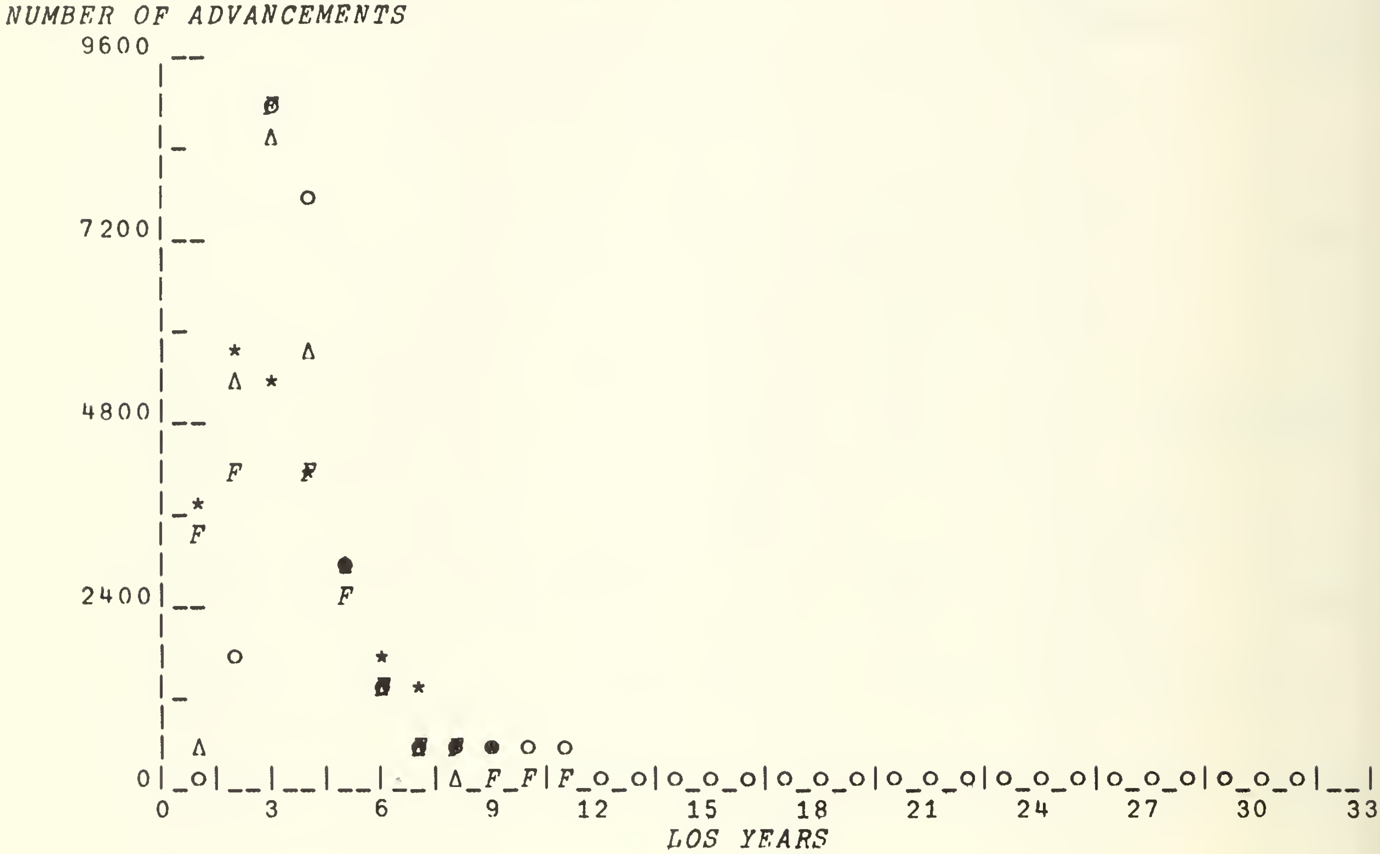
MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	13	19.08	1.98
REGRESSION	11	20.27	2.59
GAMMA DIST	13	19.38	2.68
F A S T	12	20.17	2.40

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=0 PAY GRADE=E4 FISCAL YEAR=1976
*:ACTUAL Δ:REGRESSION *:GAMMA DIST.(2.28 1.06) F: F A S T*



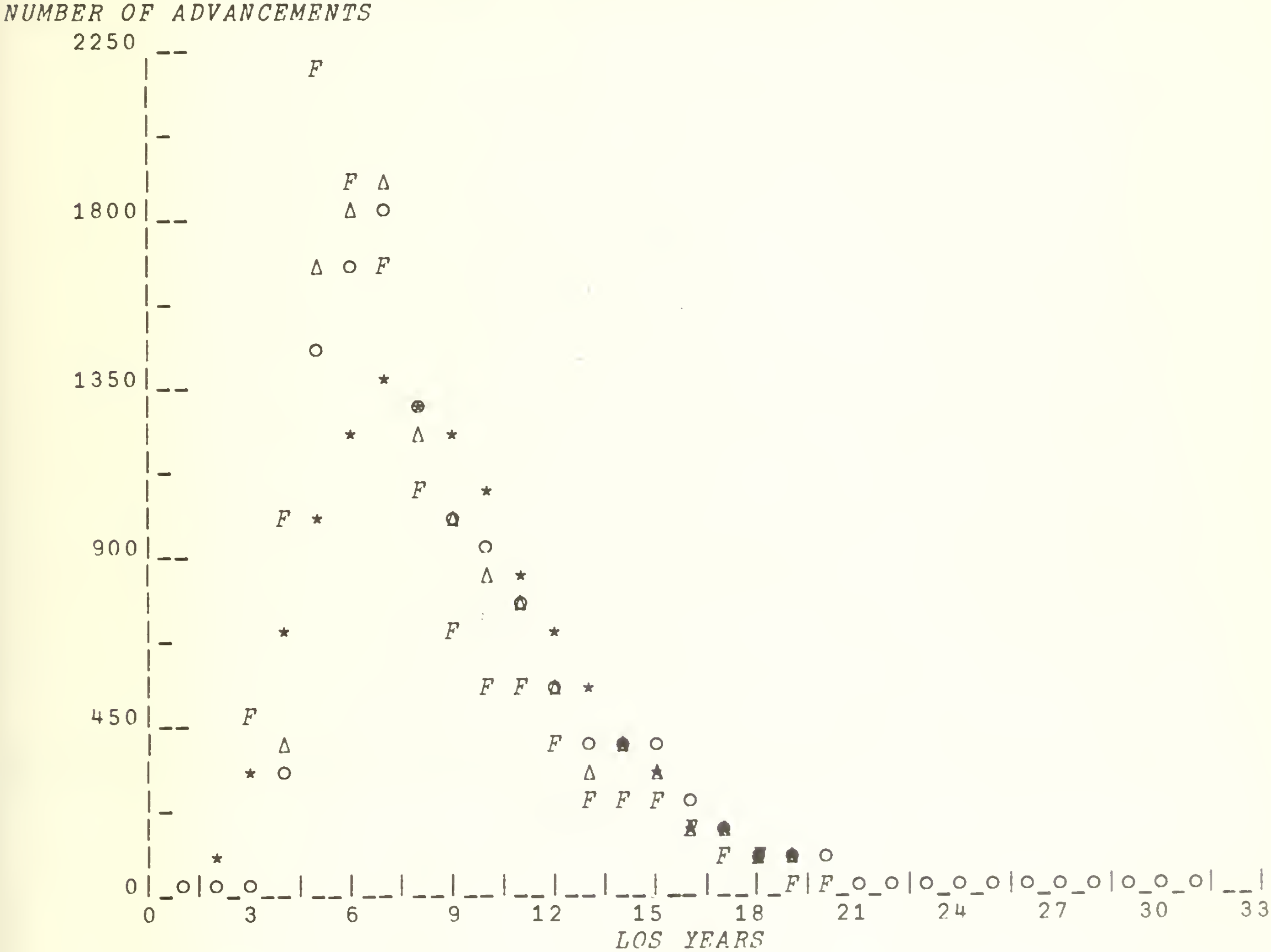
MODEL	VOLUME	MEAN LOS	ST. ERR.
CTUAL	49388	2.53	1.69
EGRESSION	49389	2.16	1.48
AMMA DIST	49388	2.25	1.40
A S T	49386	1.82	1.50

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=0 PAY GRADE=E5 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(2.67 0.72) F: F A S T



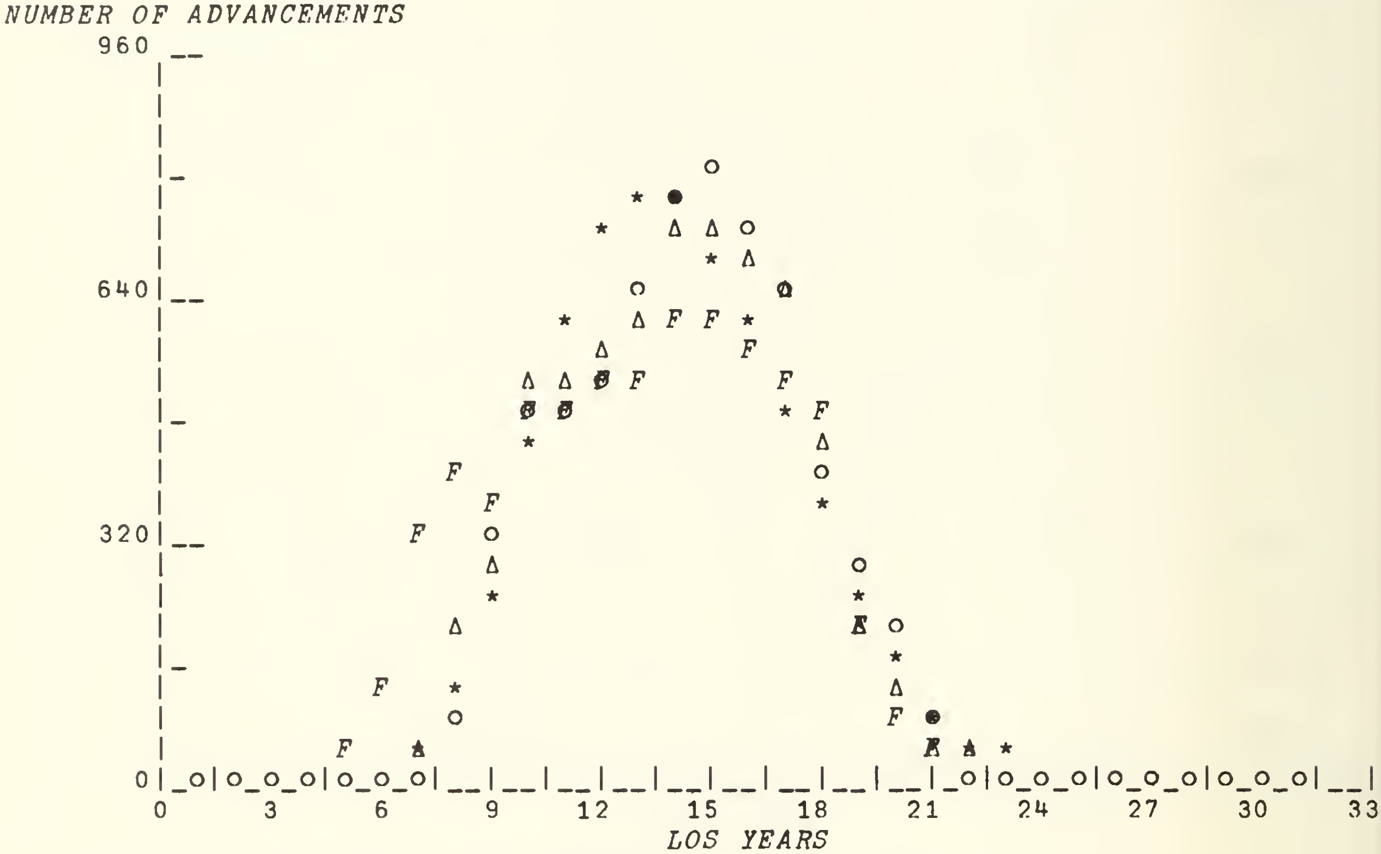
MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	27166	4.52	2.65
REGRESSION	27166	3.90	2.34
GAMMA DIST	27165	3.72	2.39
F A S T	27165	3.70	2.57

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=0 PAY GRADE=E6 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(5.45 0.61) F: F A S T



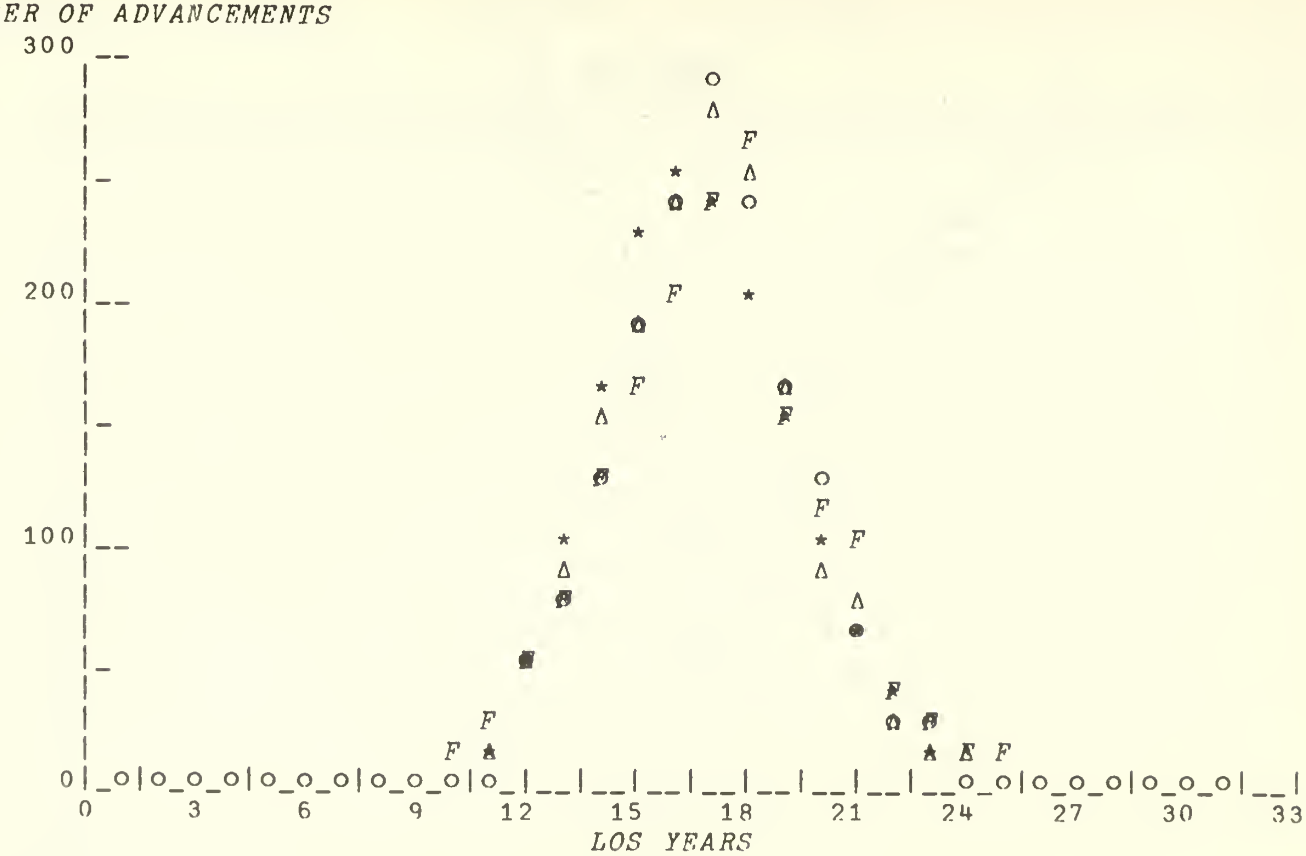
MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	12099	8.94	3.67
REGRESSION	12101	8.64	3.63
GAMMA DIST	12100	8.91	3.81
F A S T	12097	7.65	3.76

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=0 PAY GRADE=E7 FISCAL YEAR=1976
o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(16.69 1.18) F: F A S T



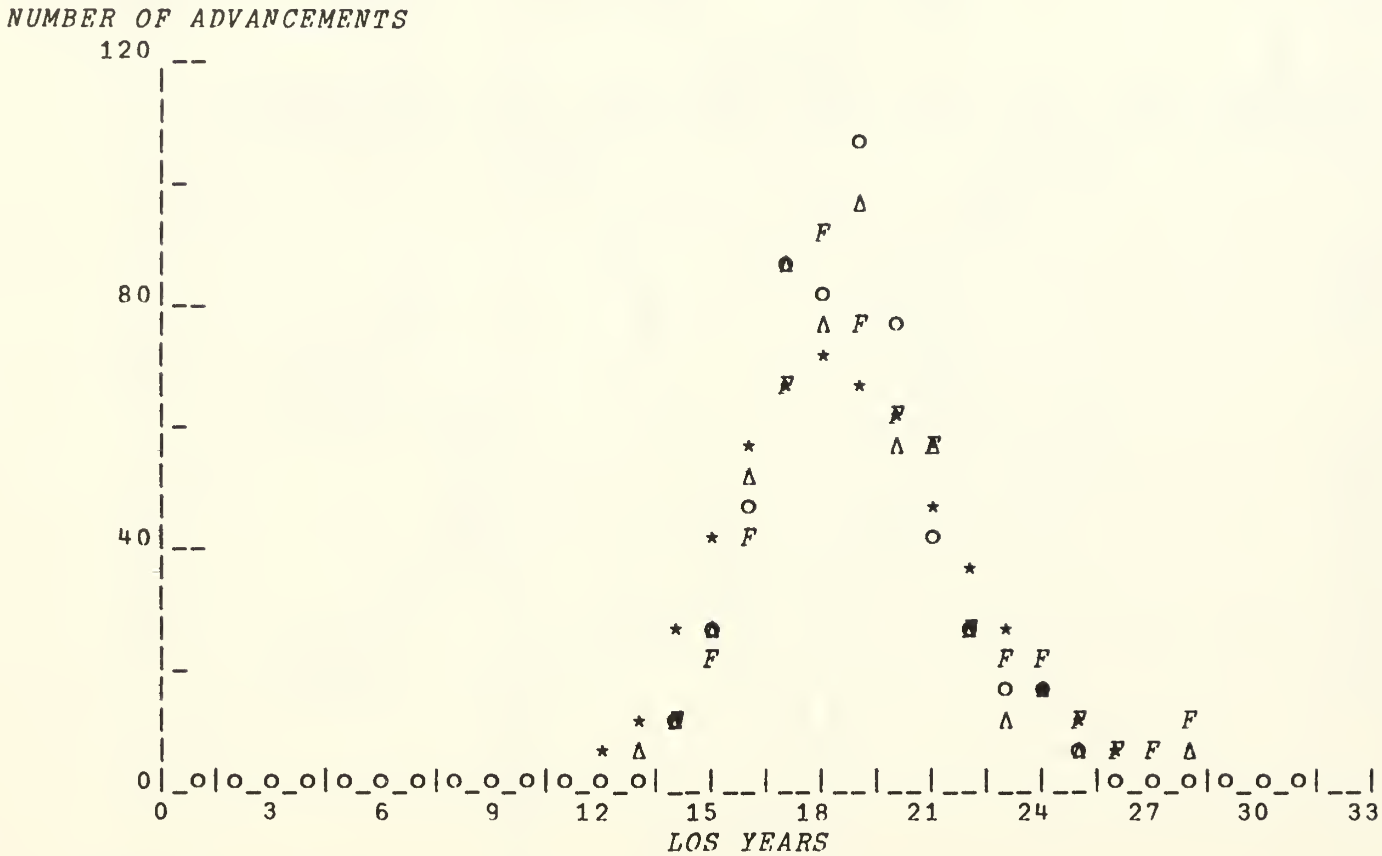
MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	6790	14.31	3.28
REGRESSION	6791	14.14	3.45
GAMMA DIST	6789	14.15	3.46
F A S T	6789	13.15	4.09

COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=0 PAY GRADE=E8 FISCAL YEAR=1976
*o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(36.24 2.17) F: F A S T*



MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	1730	16.92	2.89
REGRESSION	1729	16.80	2.95
GAMMA DIST	1732	16.71	2.78
F A S T	1730	17.13	3.42

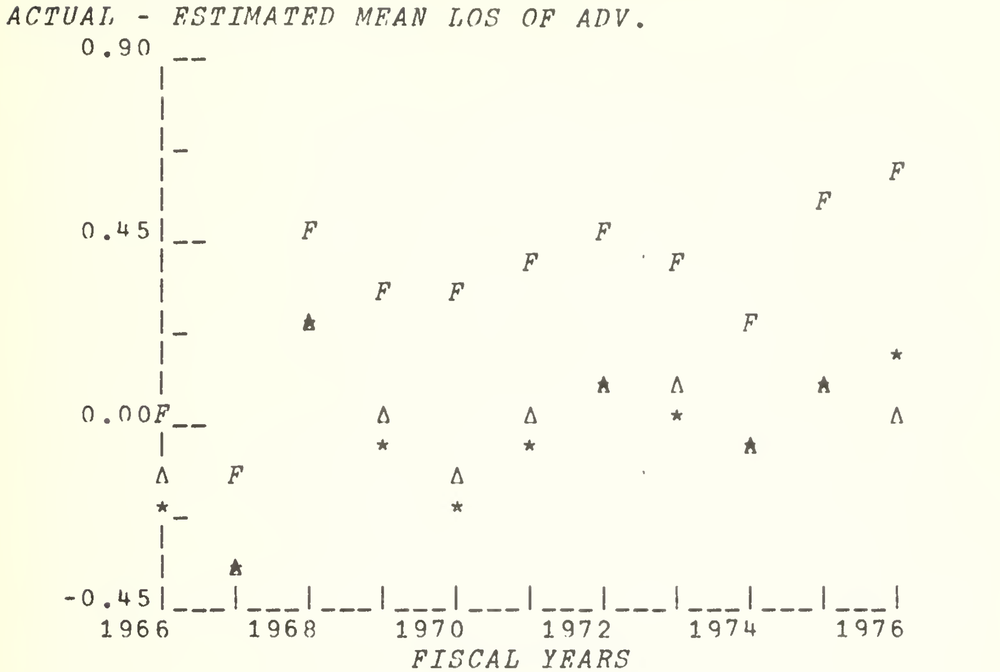
COMPARISON OF ESTIMATES OF NUMBER OF ADVANCEMENTS
RATING=0 PAY GRADE=E9 FISCAL YEAR=1976
*o:ACTUAL Δ:REGRESSION *:GAMMA DIST.(35.16 1.88) F: F A S T*

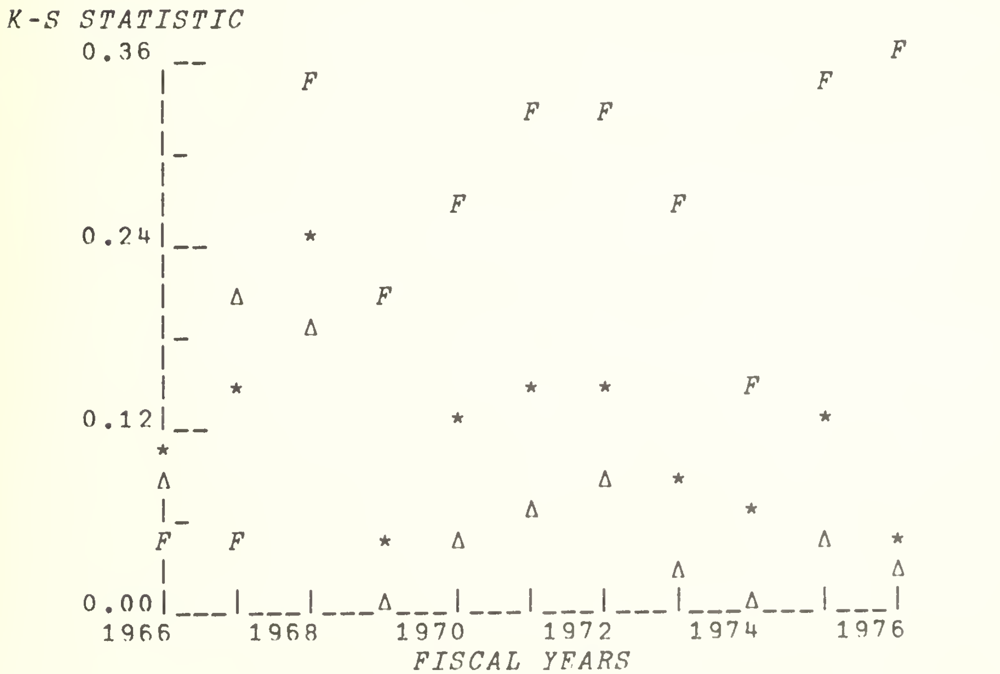


MODEL	VOLUME	MEAN LOS	ST. ERR.
ACTUAL	562	18.82	2.71
REGRESSION	561	18.81	3.13
GAMMA DIST	561	18.67	3.13
F A S T	563	19.49	3.49

APPENDIX E

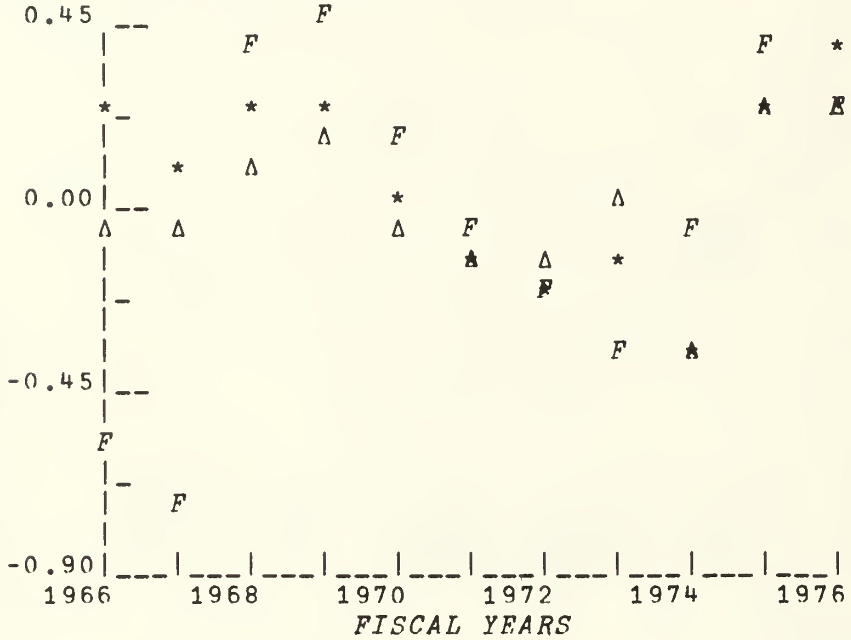
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=300 *PAY GRADE*=4
Δ: *REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*



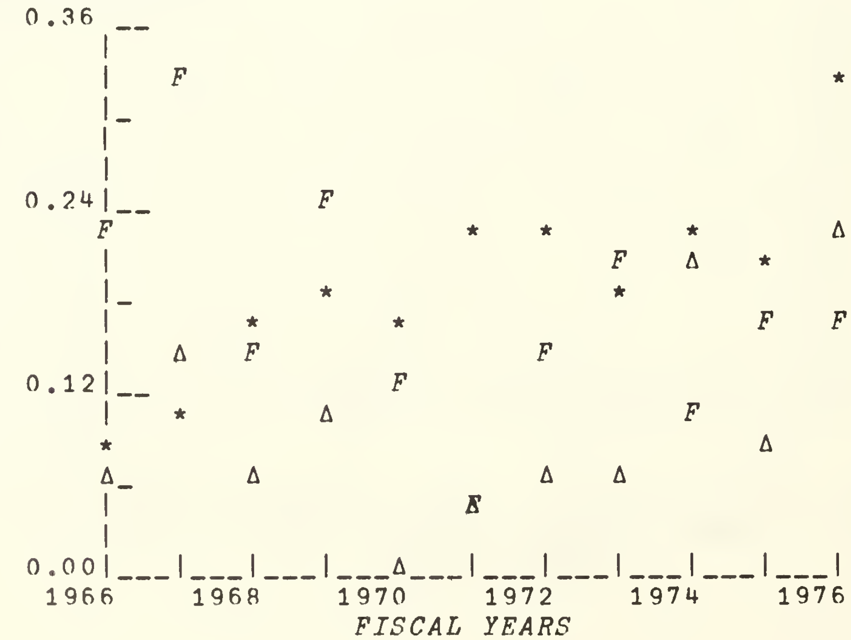


COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=300 PAY GRADE=5
*Δ: REGRESSION *: GAMMA DIST. F: F A S T*

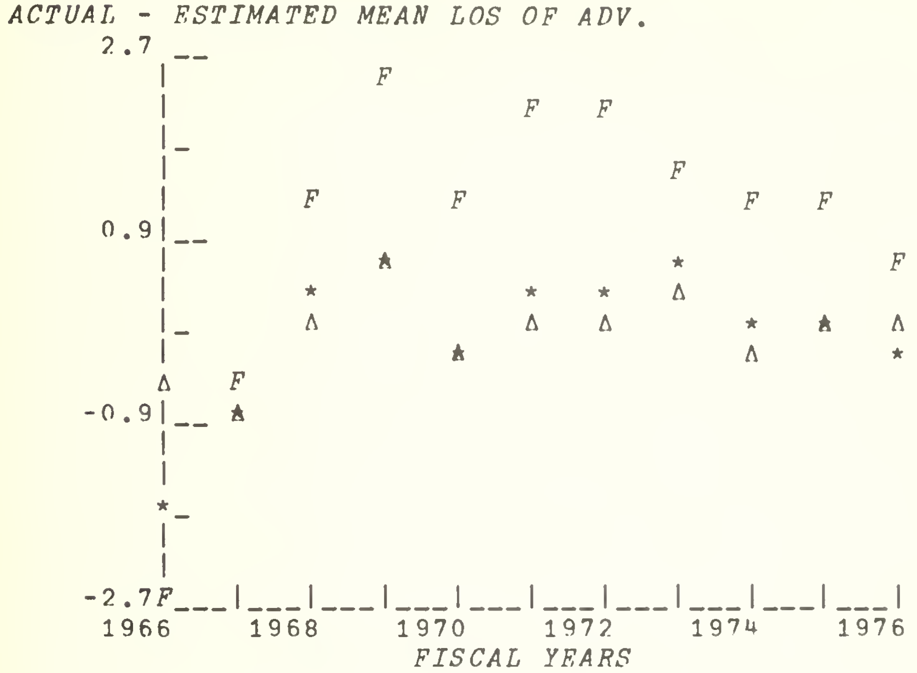
ACTUAL - ESTIMATED MEAN LOS OF ADV.

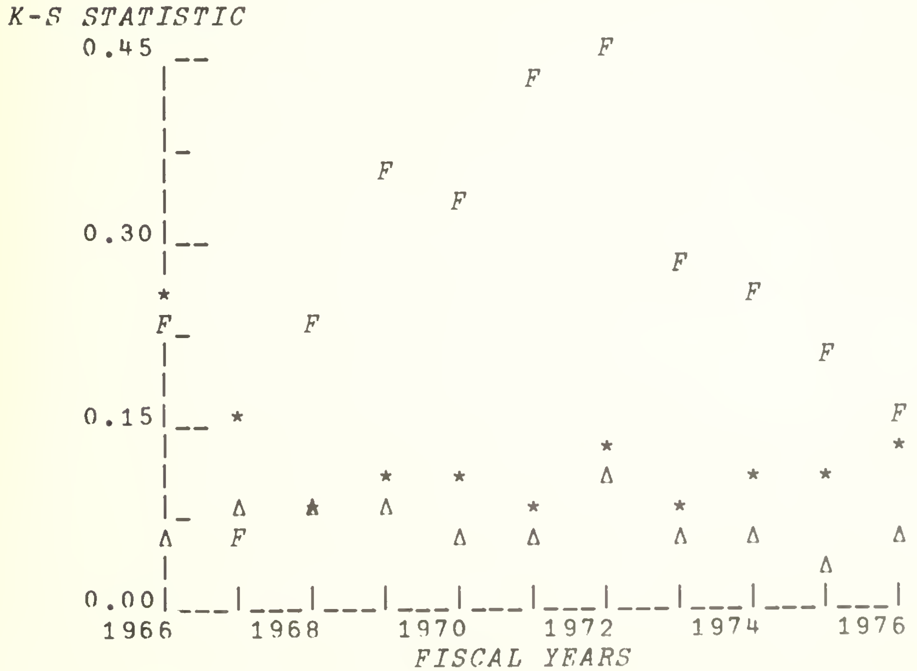


K-S STATISTIC

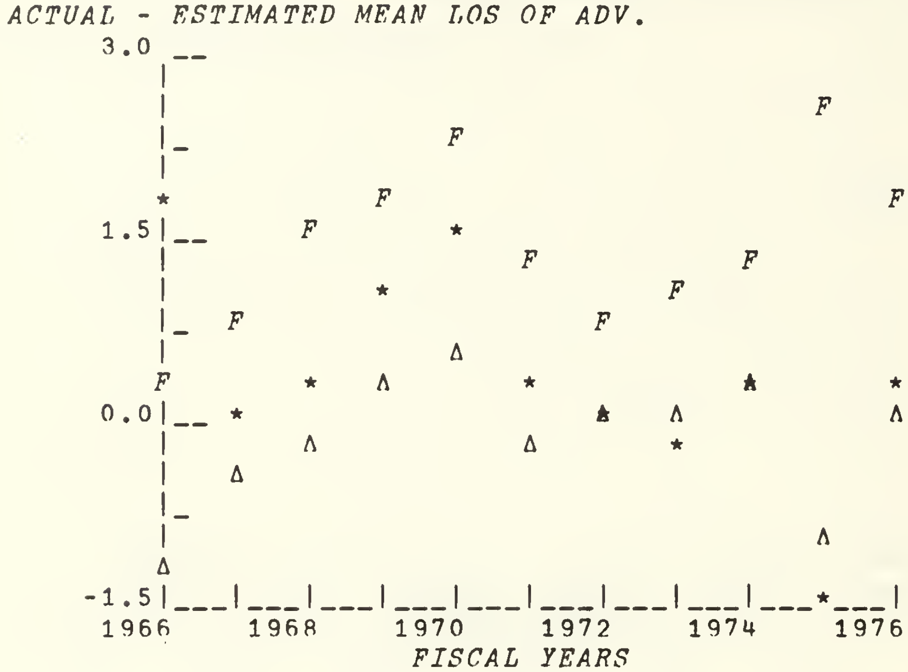


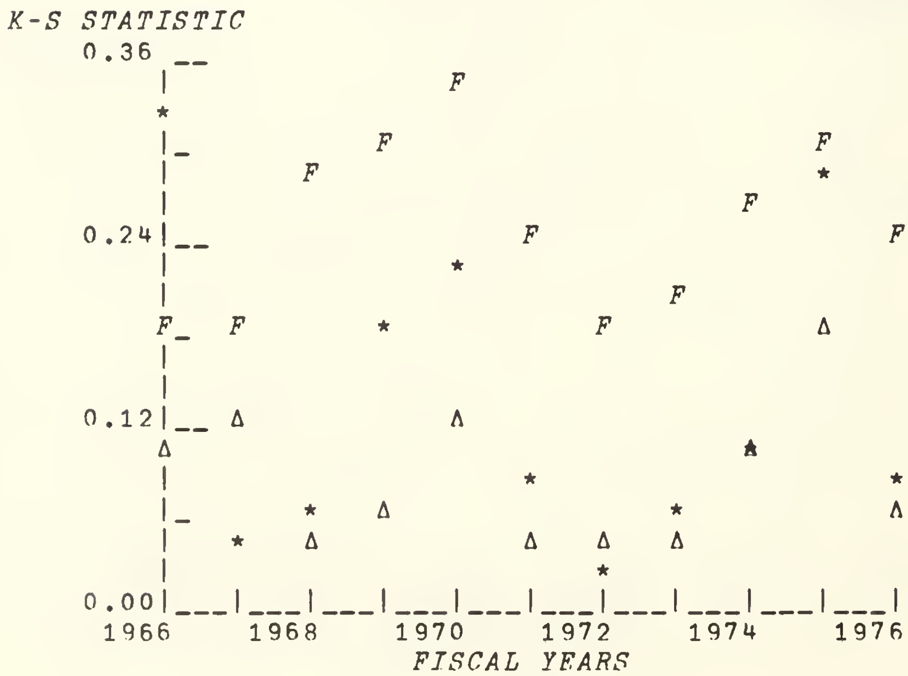
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=300 PAY GRADE=6
*Λ: REGRESSION *: GAMMA DIST. F: F A S T*



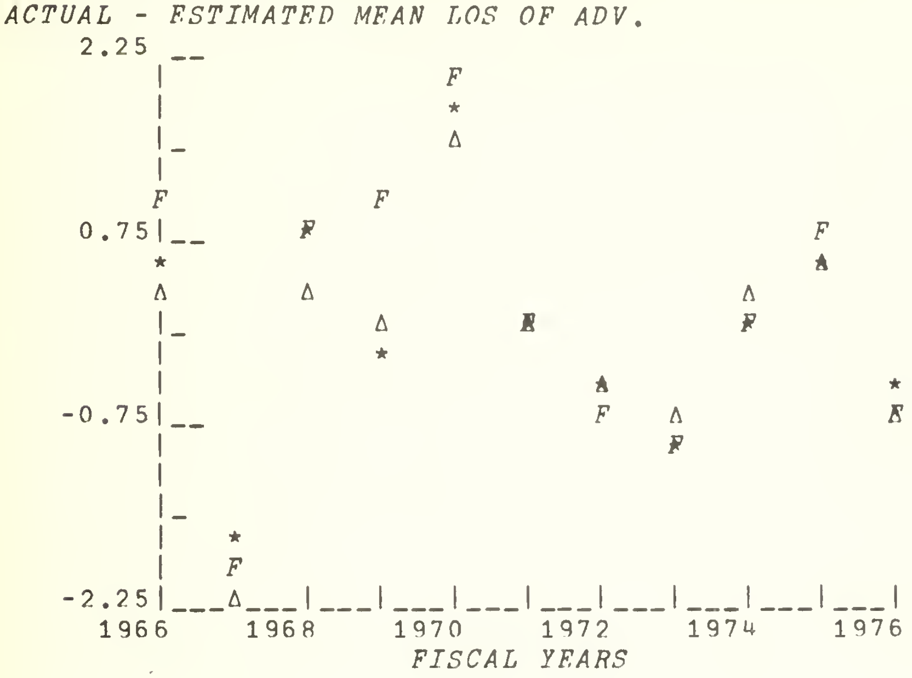


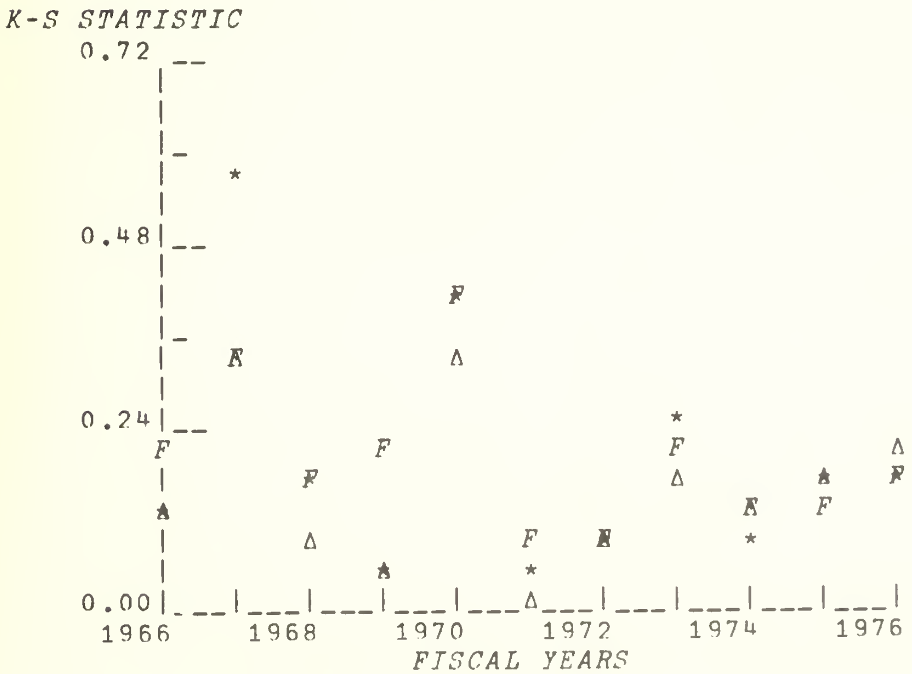
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=300 PAY GRADE=7
Δ: REGRESSION *: GAMMA DIST. F: F A S T



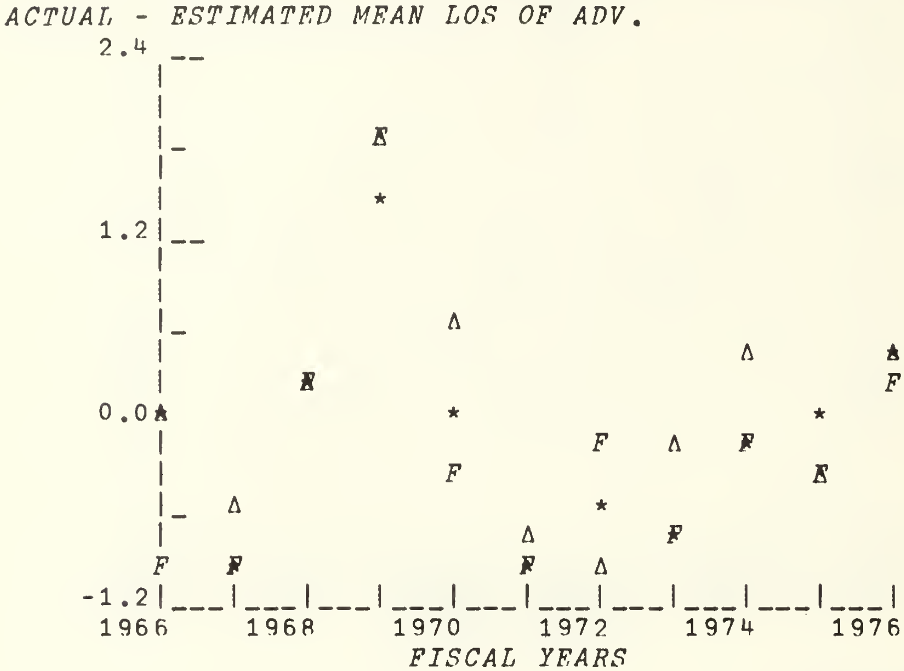


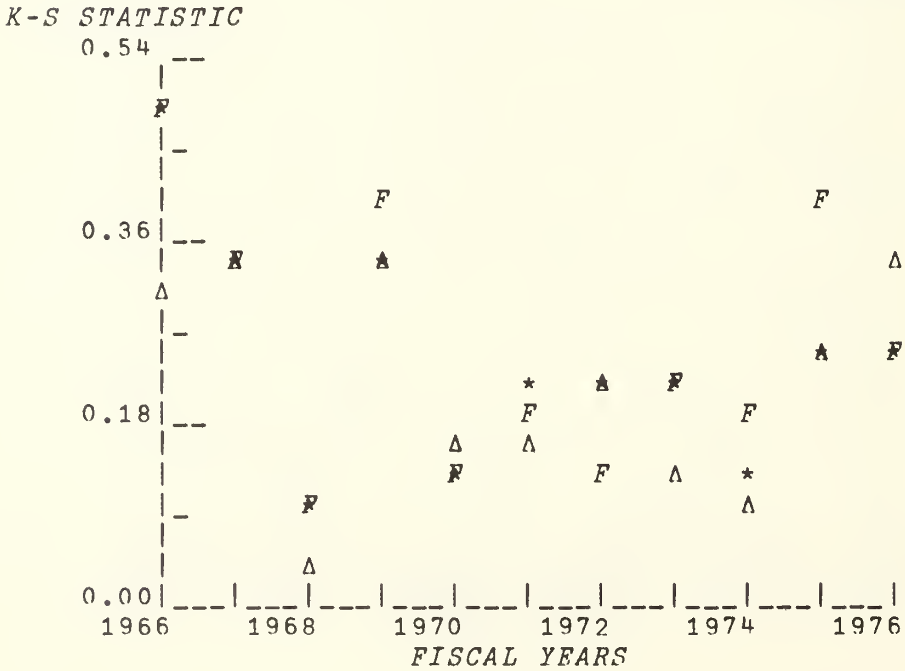
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=300 PAY GRADE=8
Δ: REGRESSION *: GAMMA DIST. F: F A S T





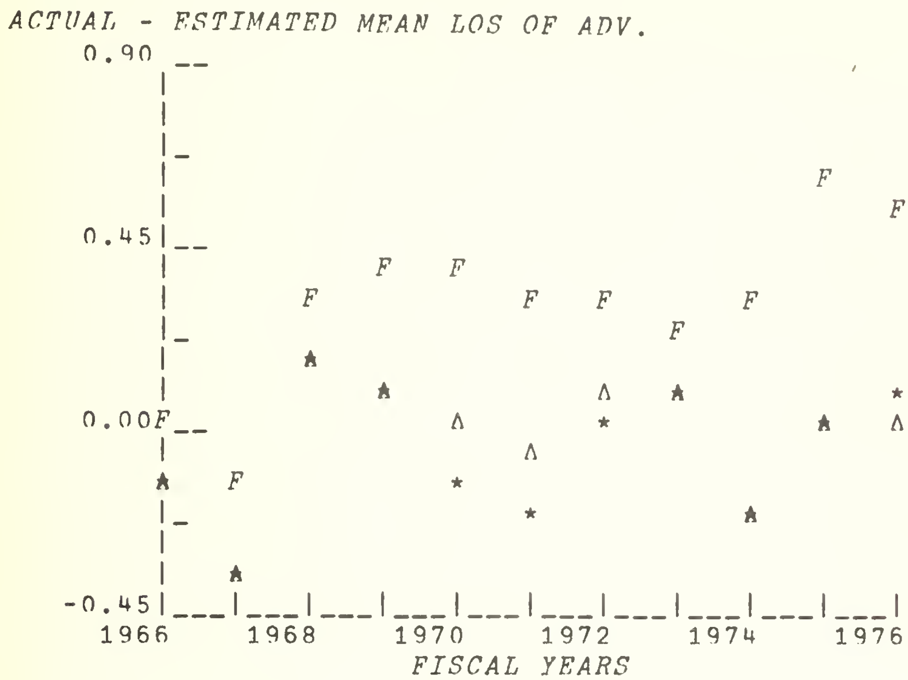
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=300 PAY GRADE=9
Λ: REGRESSION *: GAMMA DIST. F: F A S T

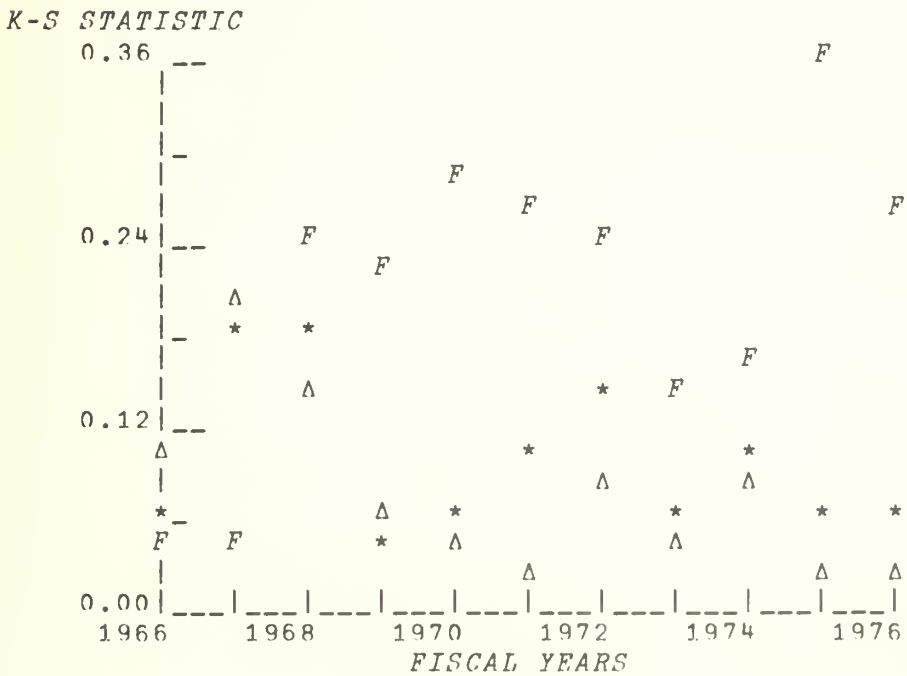




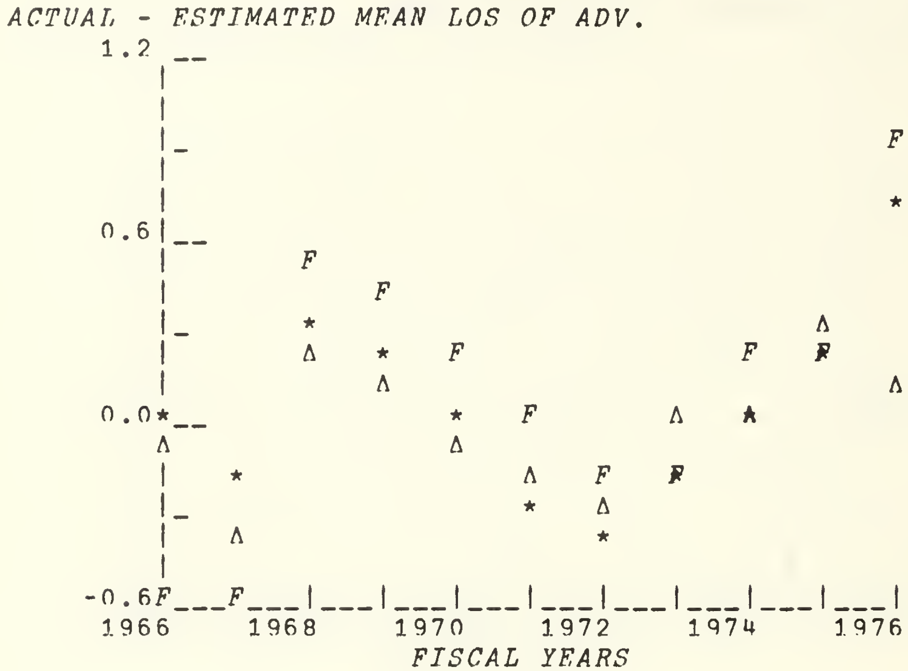
APPENDIX E (cont'd)

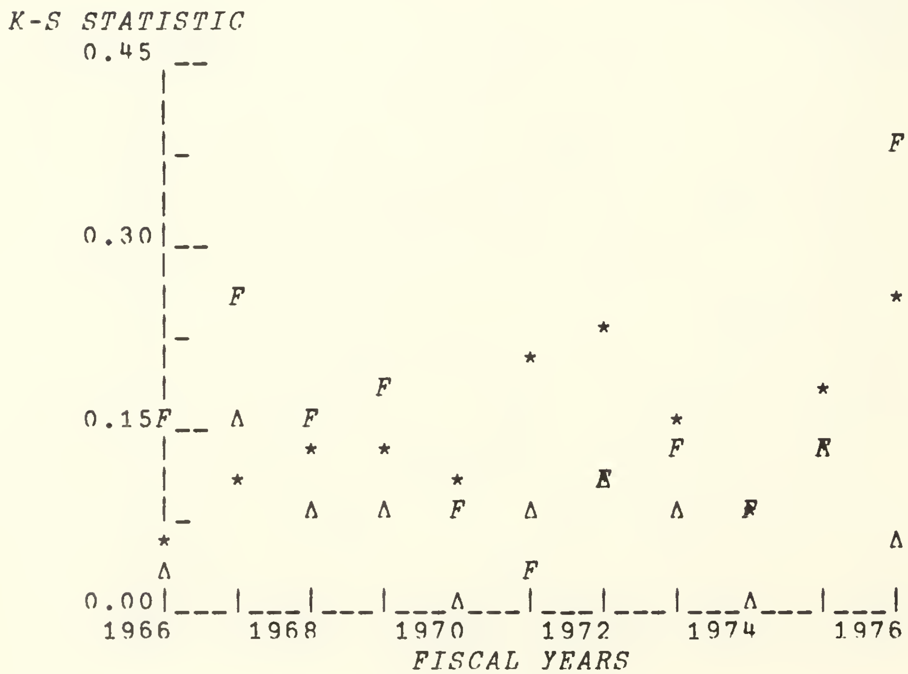
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1500 PAY GRADE=4
Λ: REGRESSION *: GAMMA DIST. F: F A S T



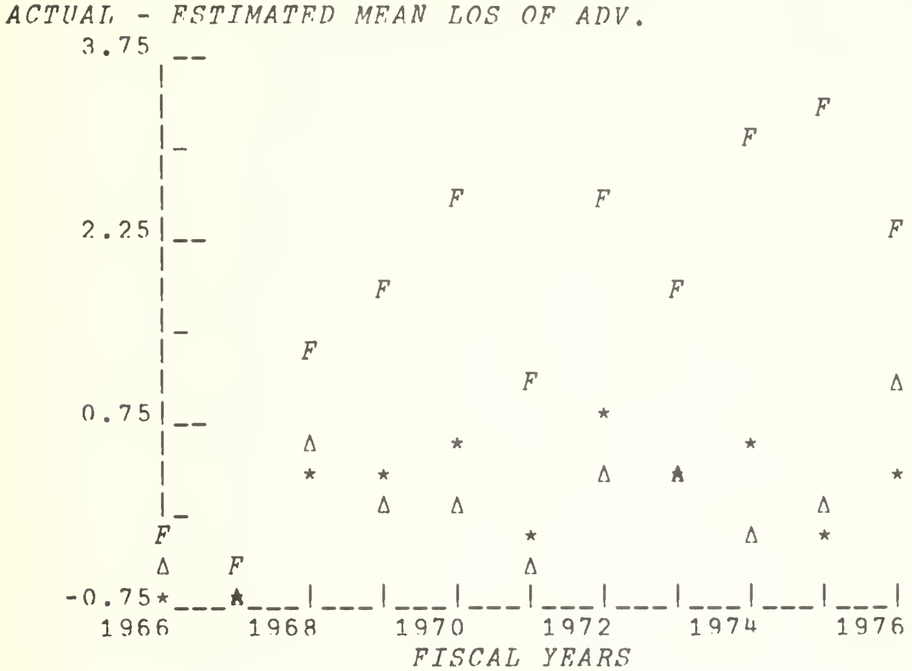


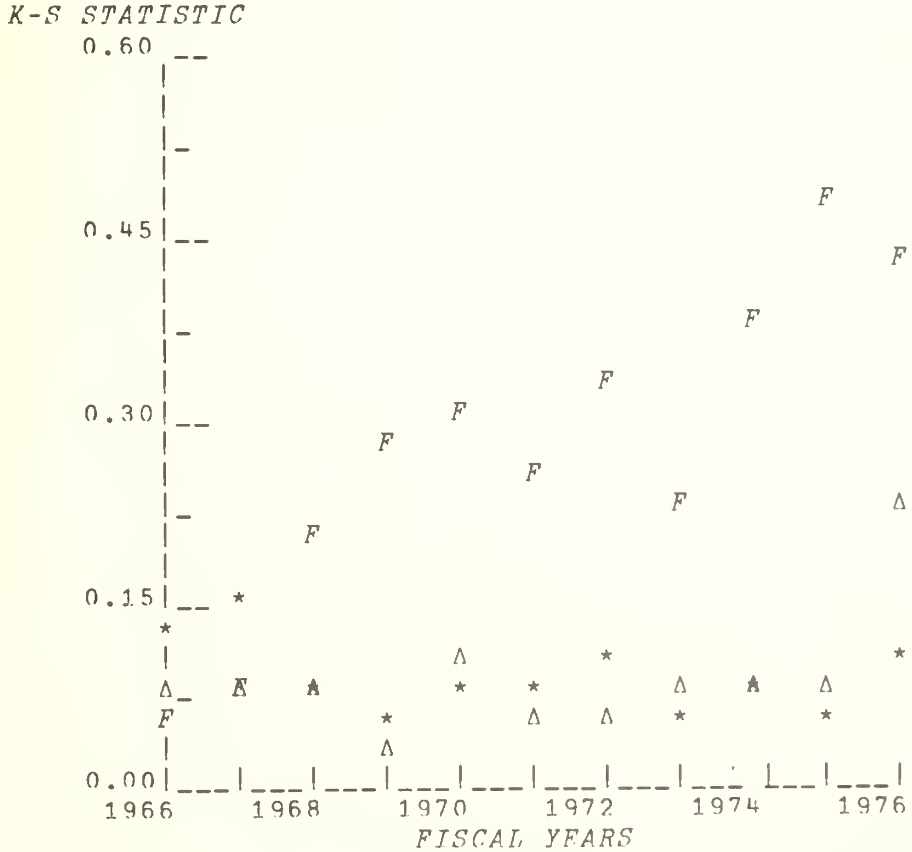
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1500 PAY GRADE=5
*Δ: REGRESSION *: GAMMA DIST. F: F A S T*





COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1500 PAY GRADE=6
*Δ: REGRESSION *: GAMMA DIST. F: F A S T*

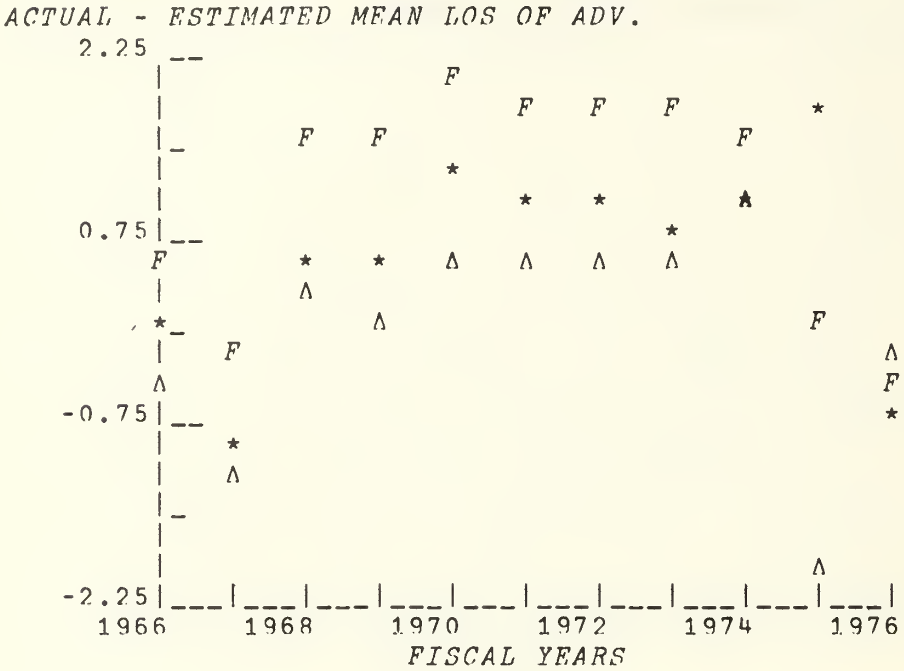


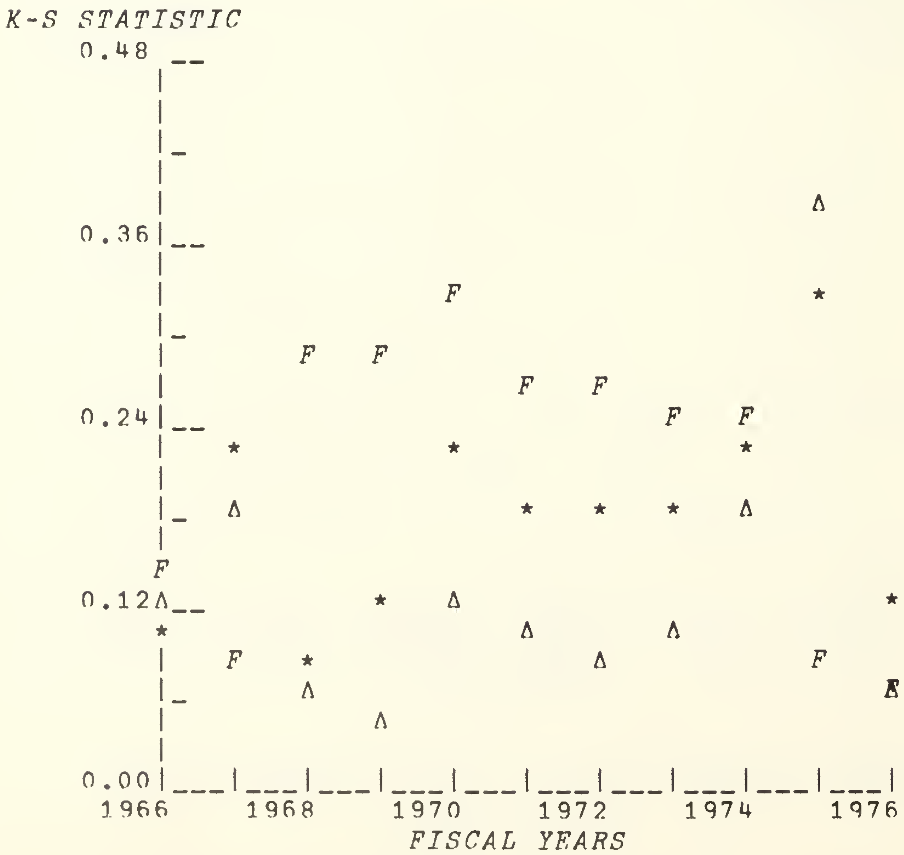


COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS

RATING=1500 PAY GRADE=7

Λ: *REGRESSION* ★: *GAMMA DIST.* *F: F A S T*



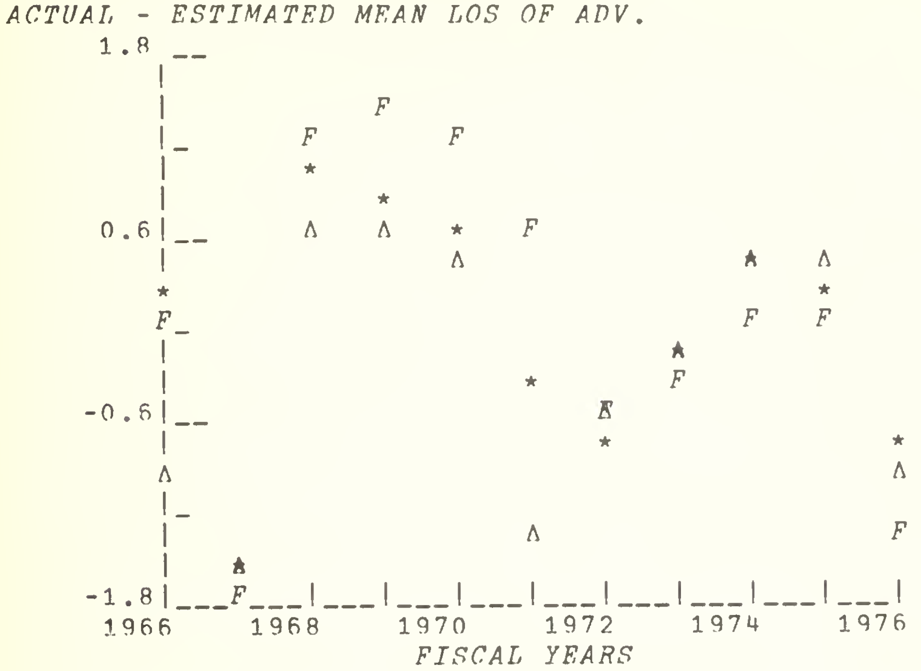


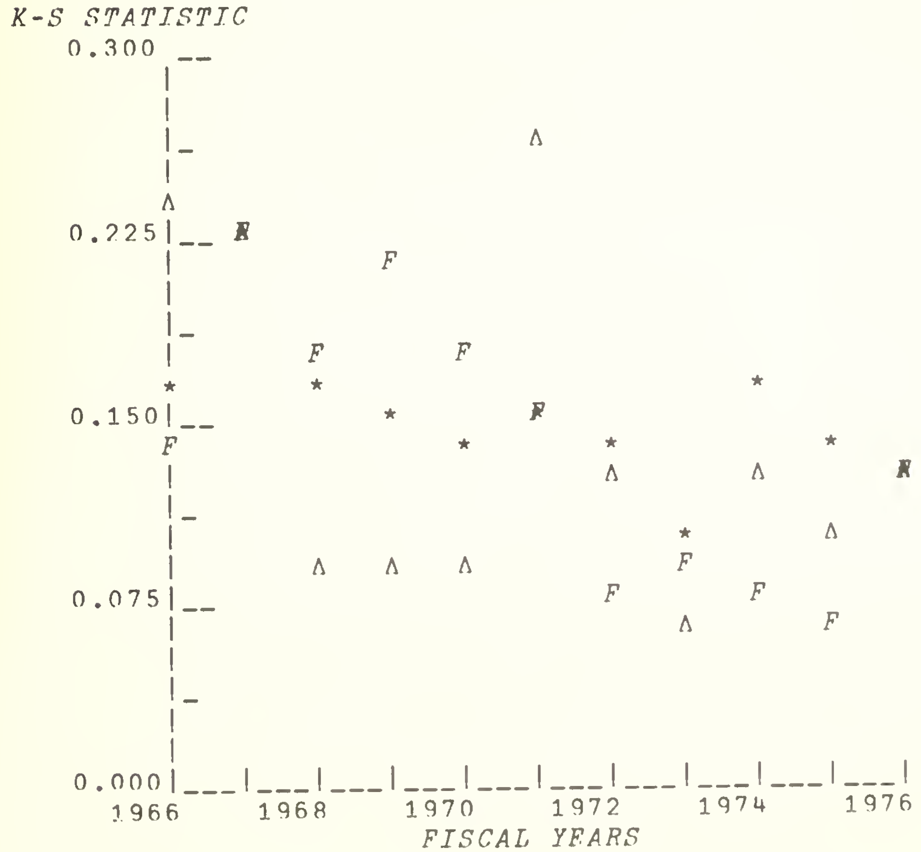
APPENDIX E (cont'd)

COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS

RATING=1500 *PAY GRADE*=8

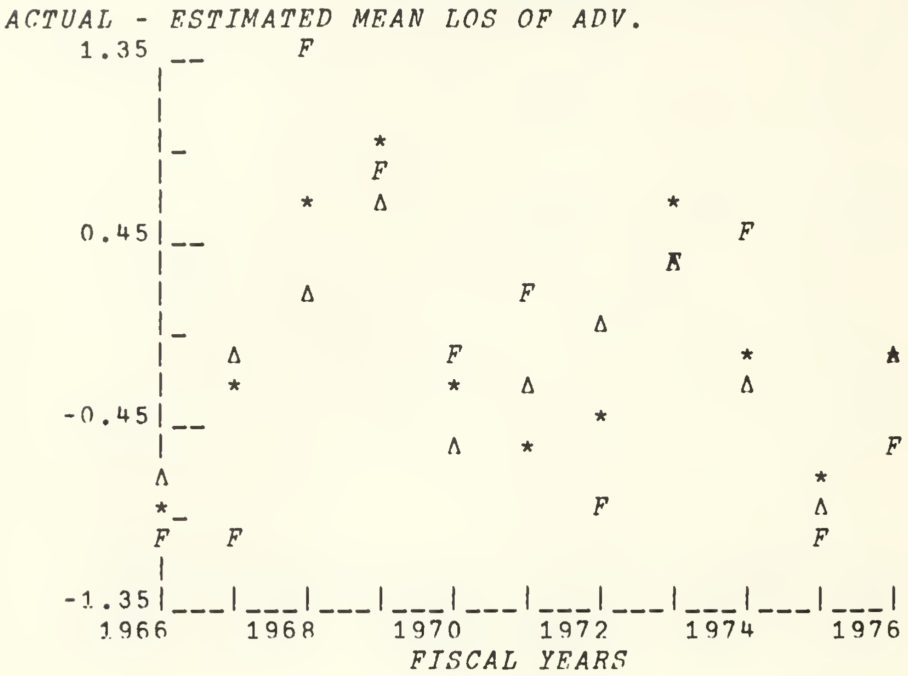
Λ: *REGRESSION* ⋆: *GAMMA DIST.* *F*: *F A S T*

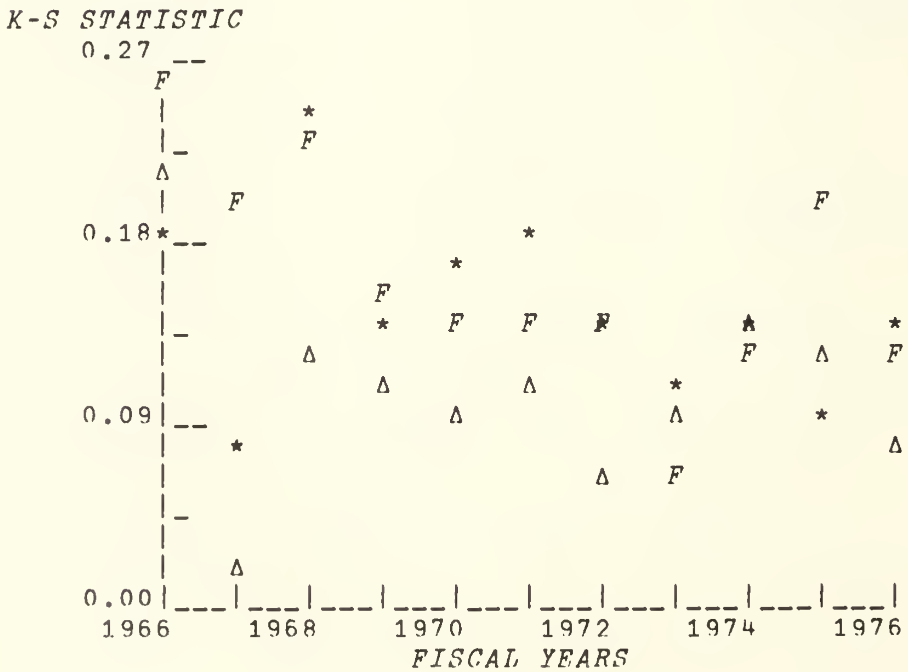




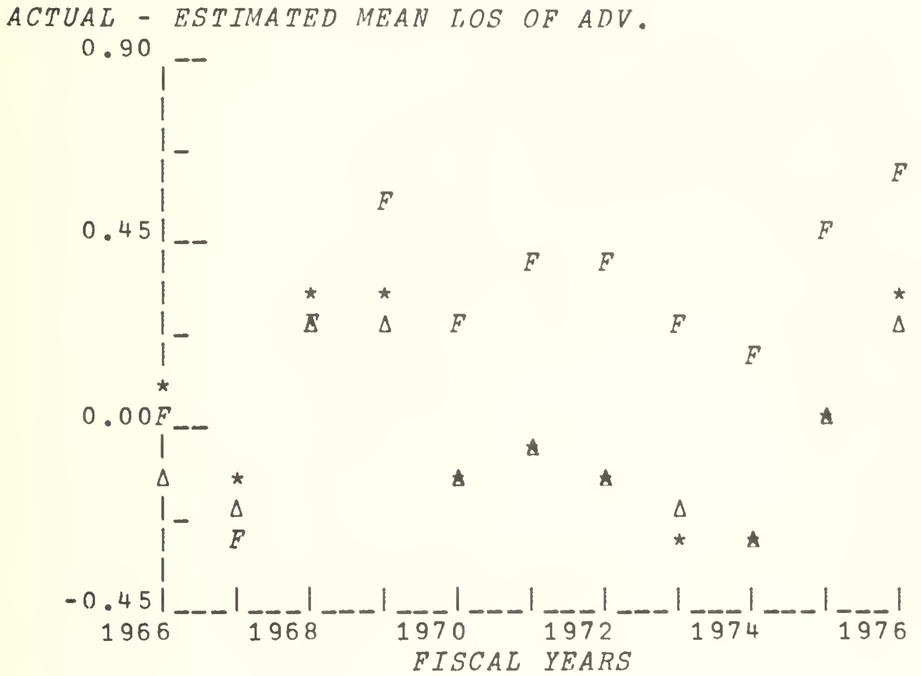
APPENDIX E (cont'd)

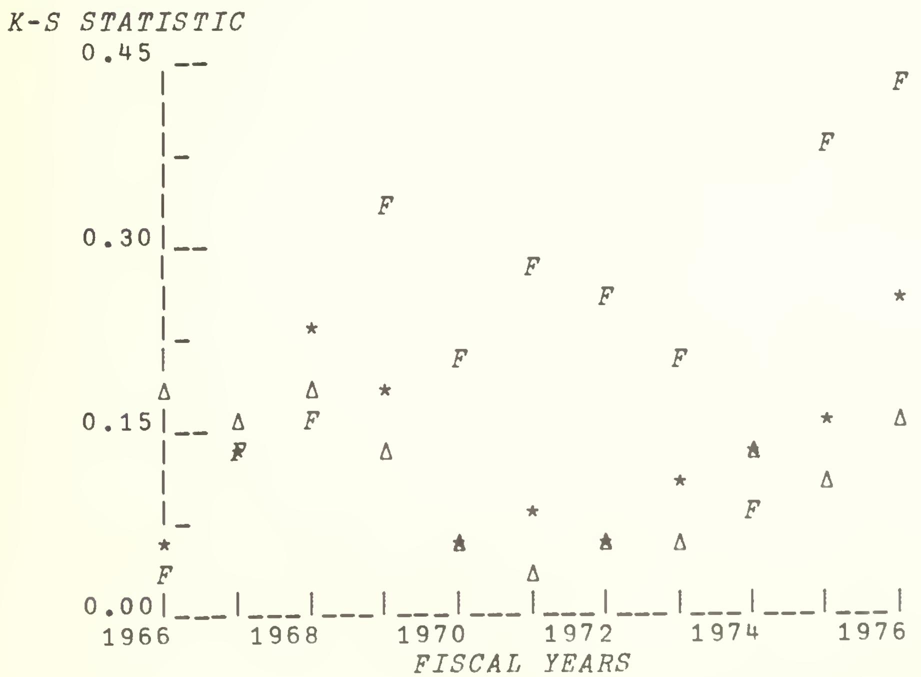
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1500 PAY GRADE=9
*Δ: REGRESSION *: GAMMA DIST. F: F A S T*



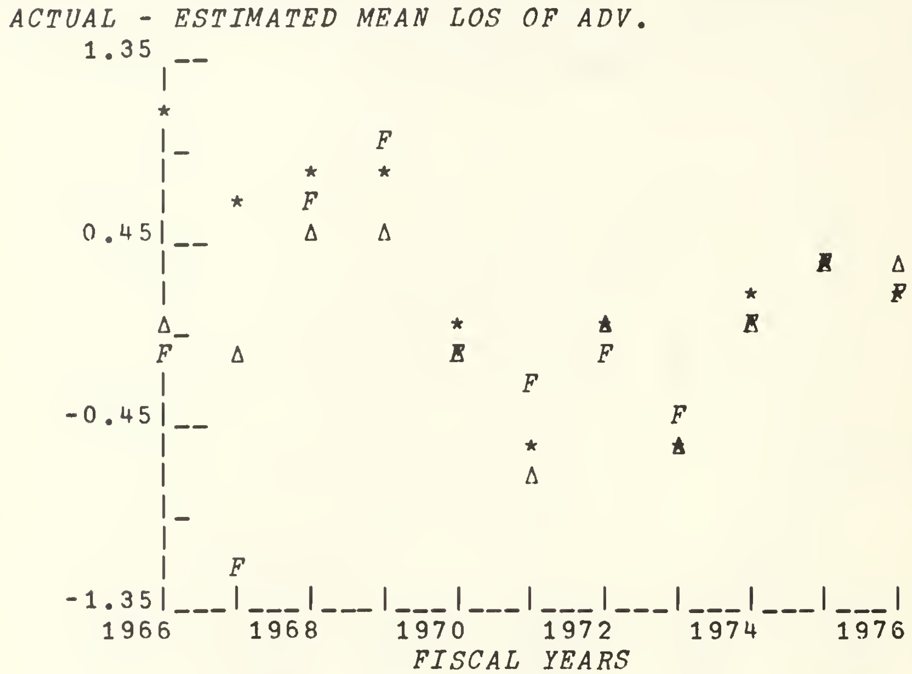


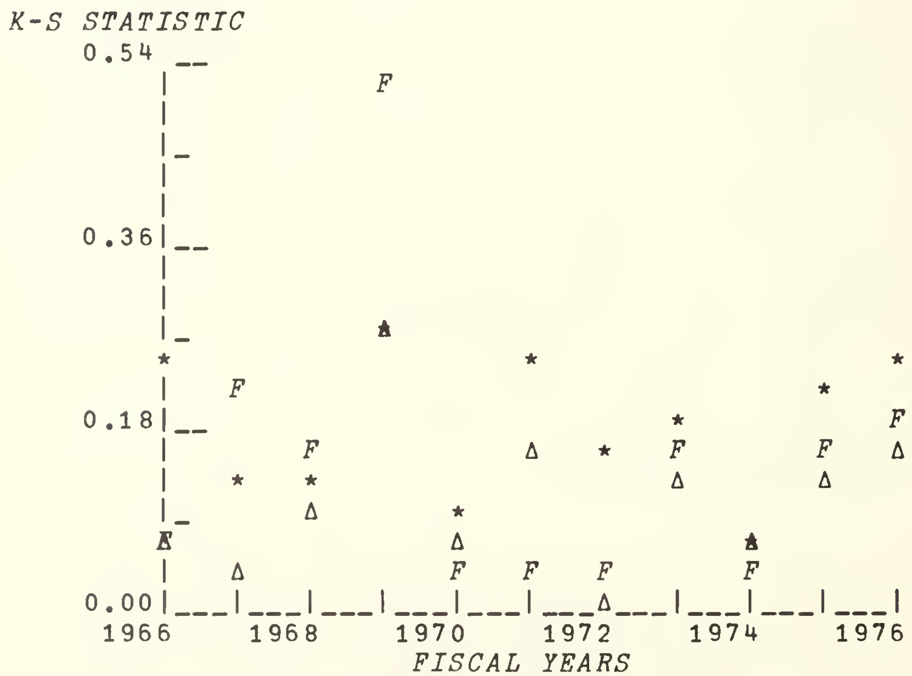
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1800 PAY GRADE=4
*Δ: REGRESSION *: GAMMA DIST. F: F A S T*





COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1800 *PAY GRADE*=5
Δ: *REGRESSION* ⋆: *GAMMA DIST.* *F*: *F A S T*



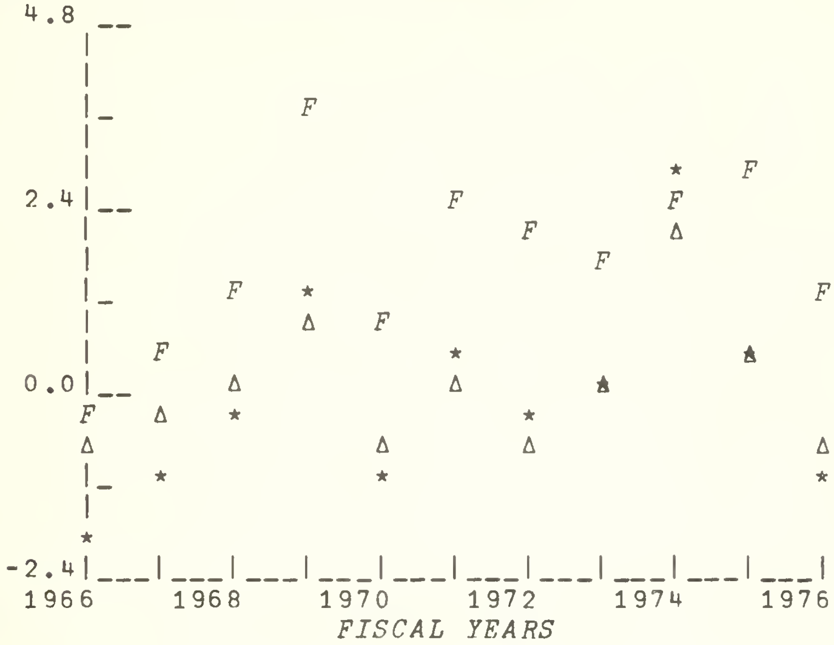


COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS

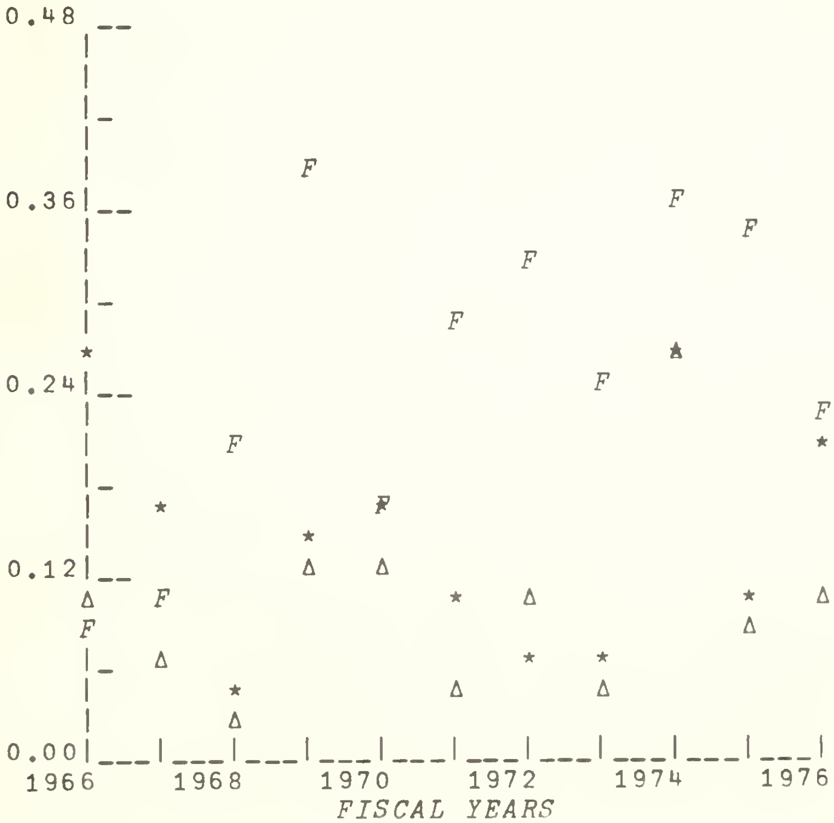
RATING=1800 *PAY GRADE*=6

Δ: *REGRESSION* *: *GAMMA DIST.* *F: F A S T*

ACTUAL - ESTIMATED MEAN LOS OF ADV.

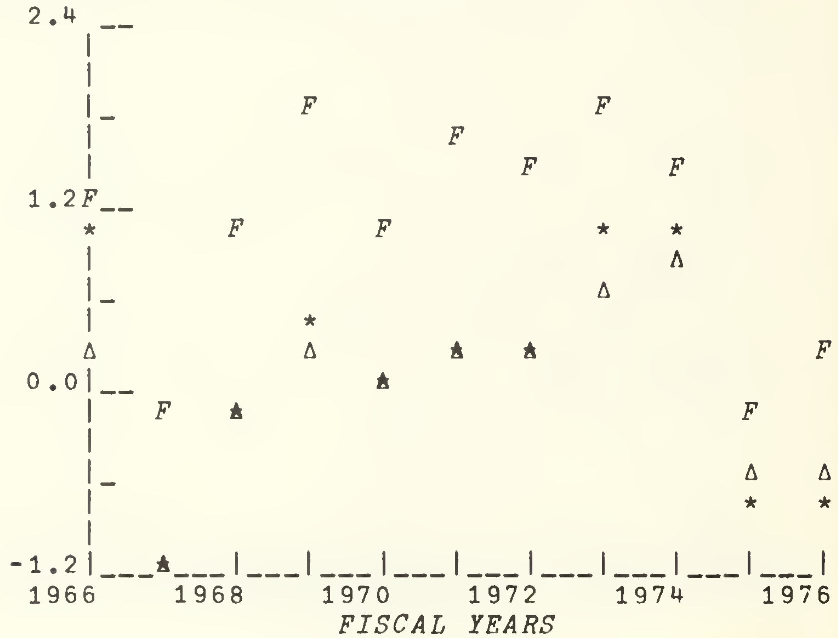


K-S STATISTIC

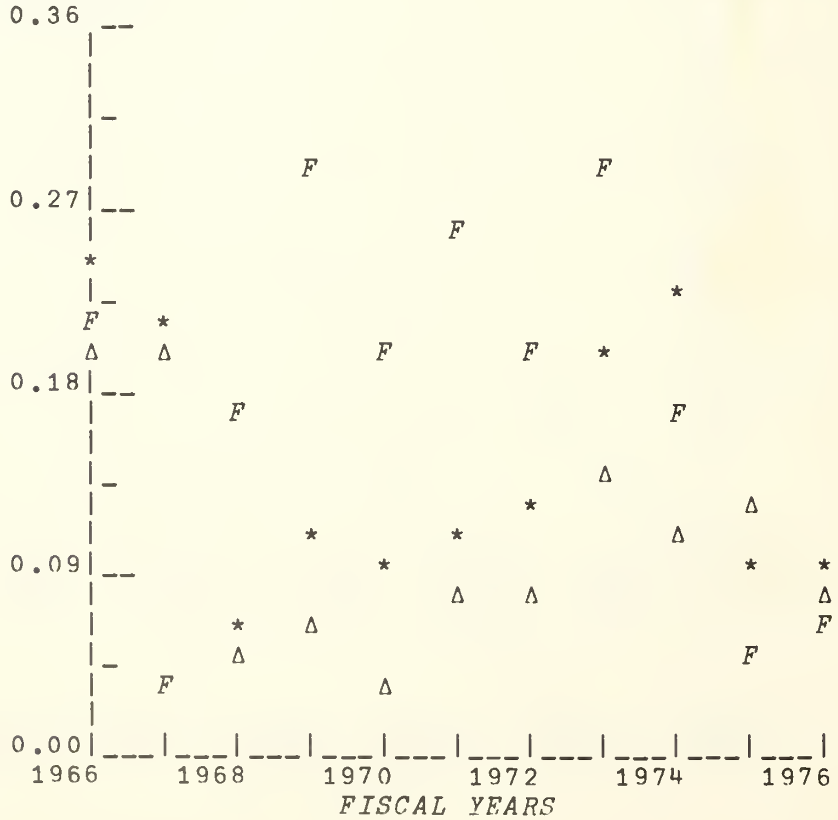


COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1800 PAY GRADE=7
*Δ: REGRESSION *: GAMMA DIST. F: F A S T*

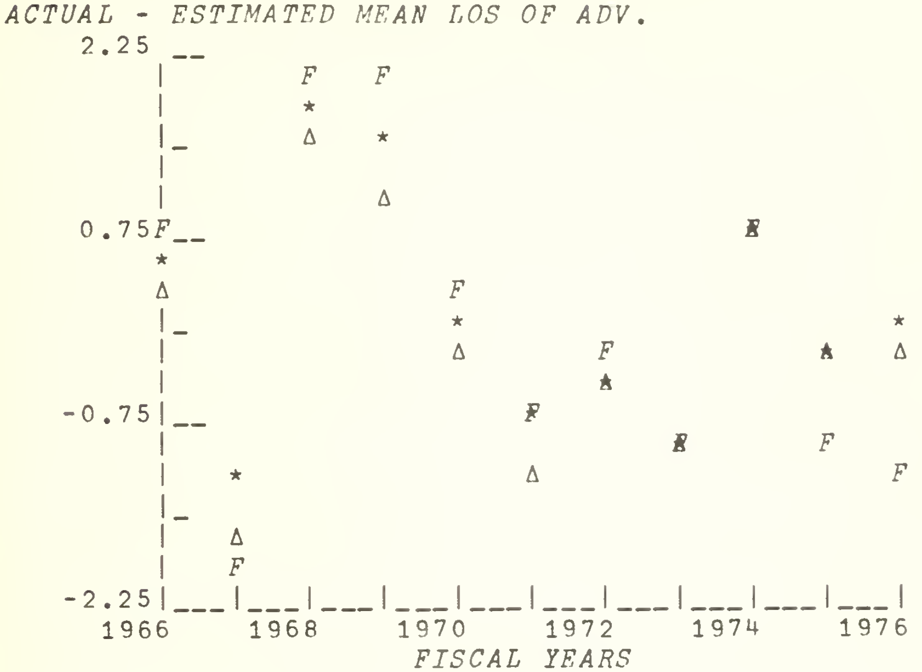
ACTUAL - ESTIMATED MEAN LOS OF ADV.

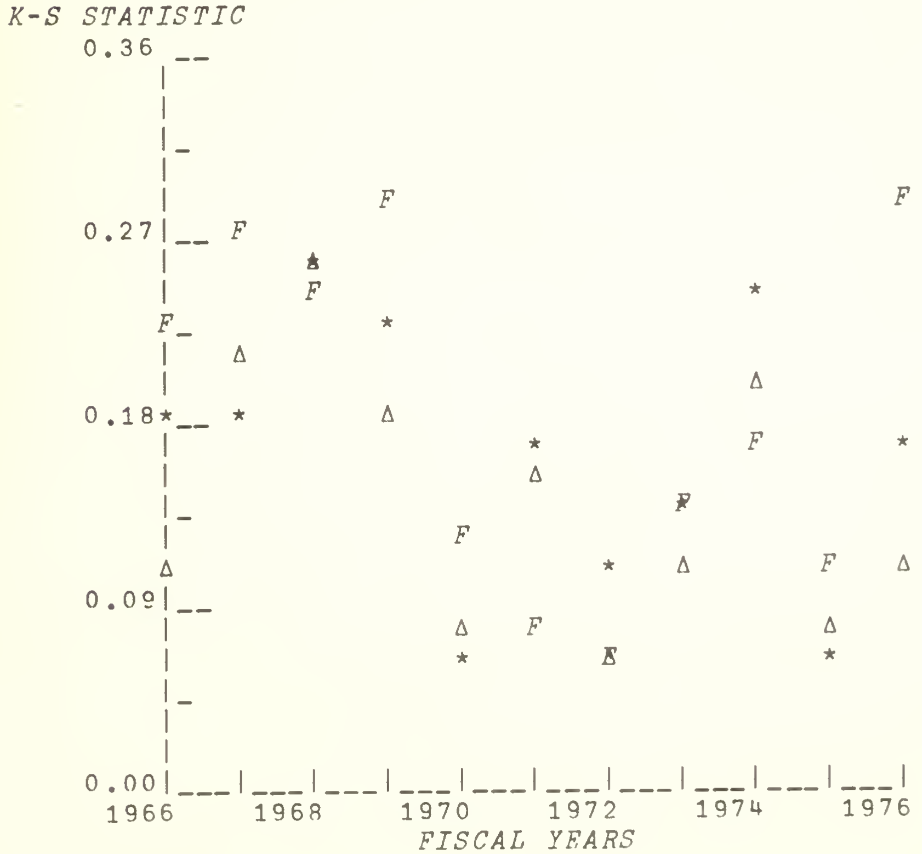


K-S STATISTIC

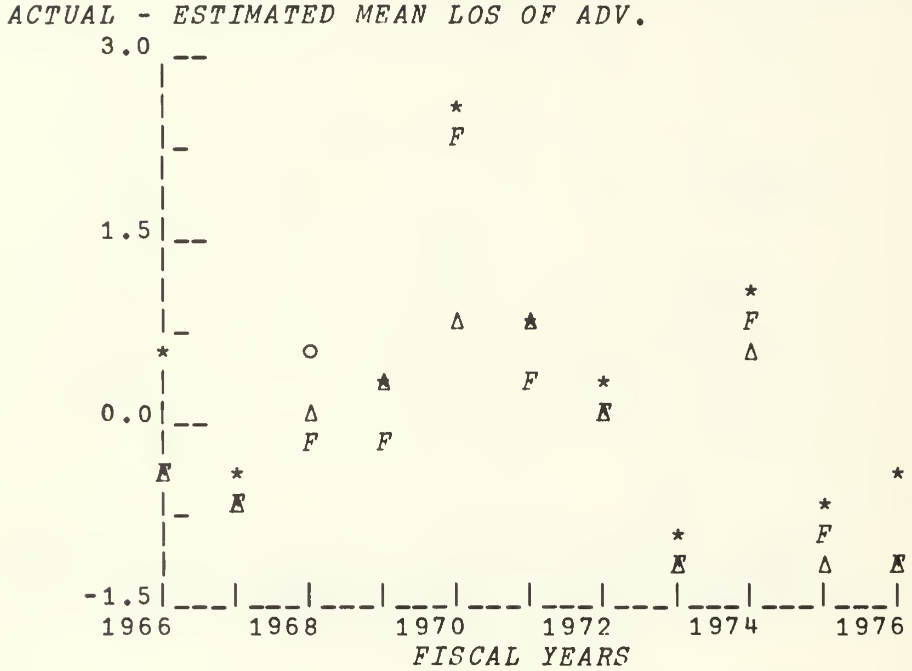


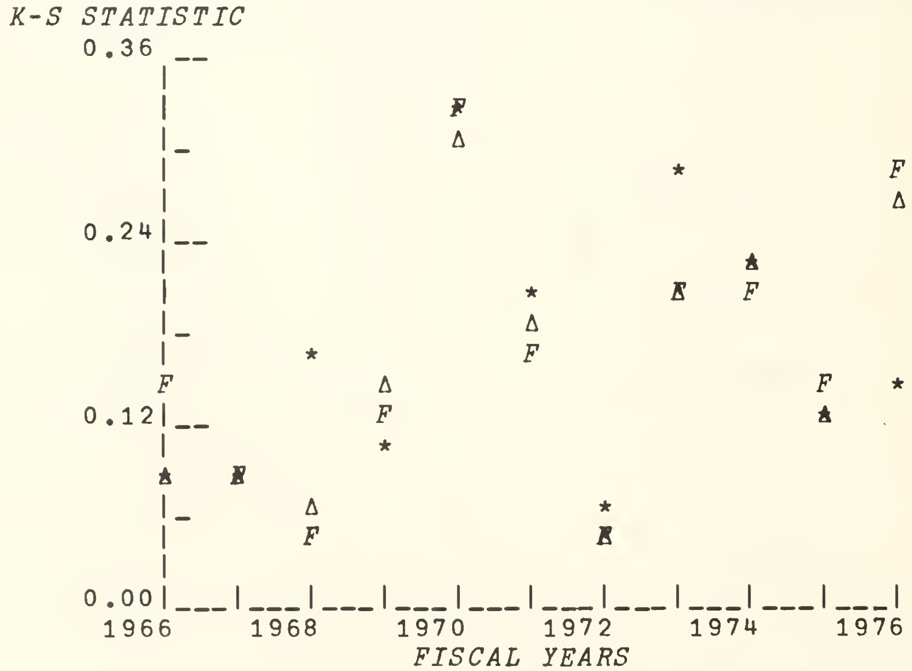
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1800 PAY GRADE=8
Δ: REGRESSION ★: GAMMA DIST. F: F A S T





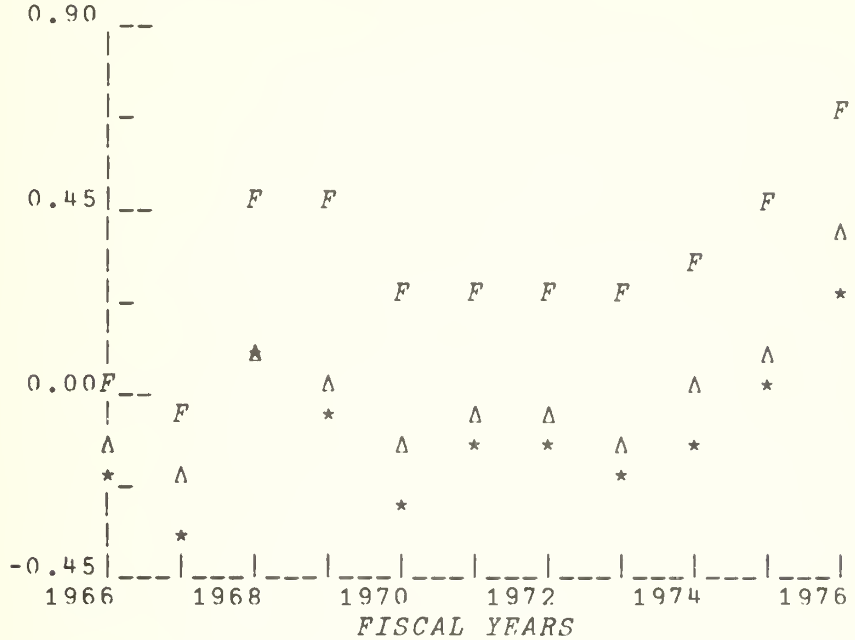
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=1800 *PAY GRADE*=9
Δ: *REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*



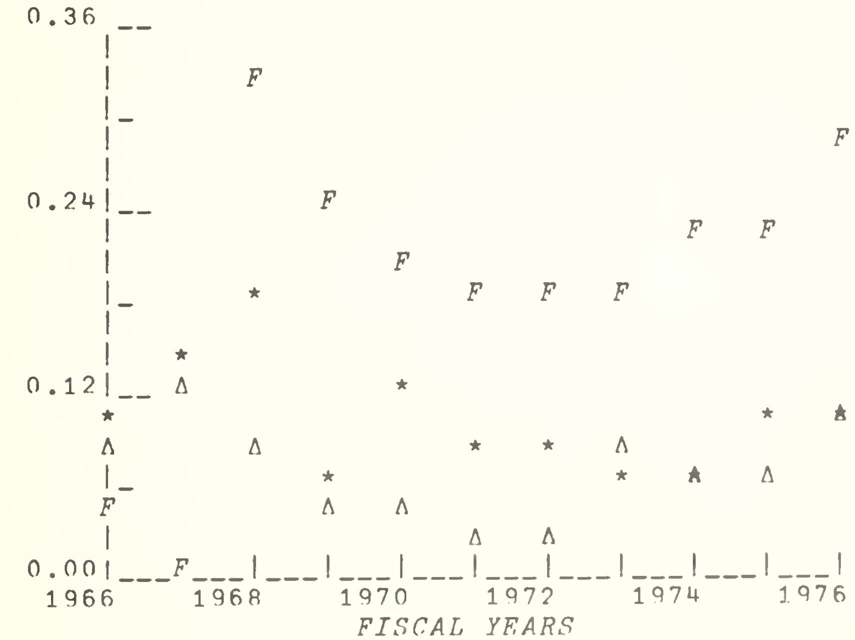


COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=0 PAY GRADE=4
*Λ: REGRESSION *: GAMMA DIST. F: F A S T*

ACTUAL - ESTIMATED MEAN LOS OF ADV.



K-S STATISTIC

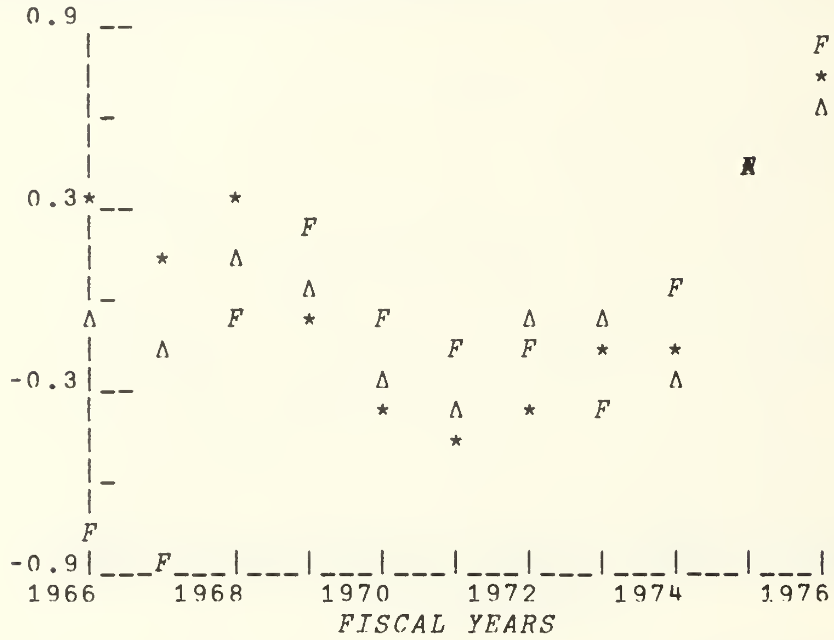


COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS

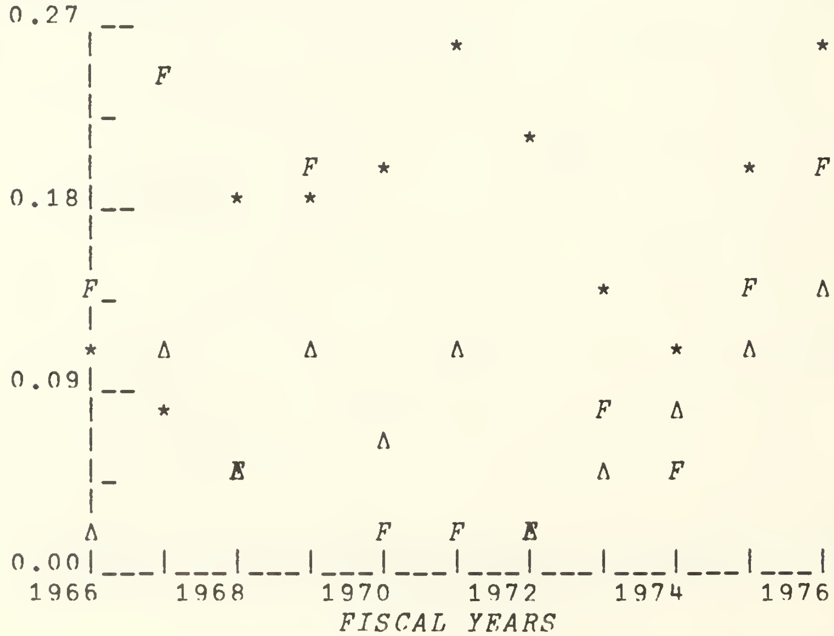
RATING=0 PAY GRADE=5

*Δ: REGRESSION *: GAMMA DIST. F: F A S T*

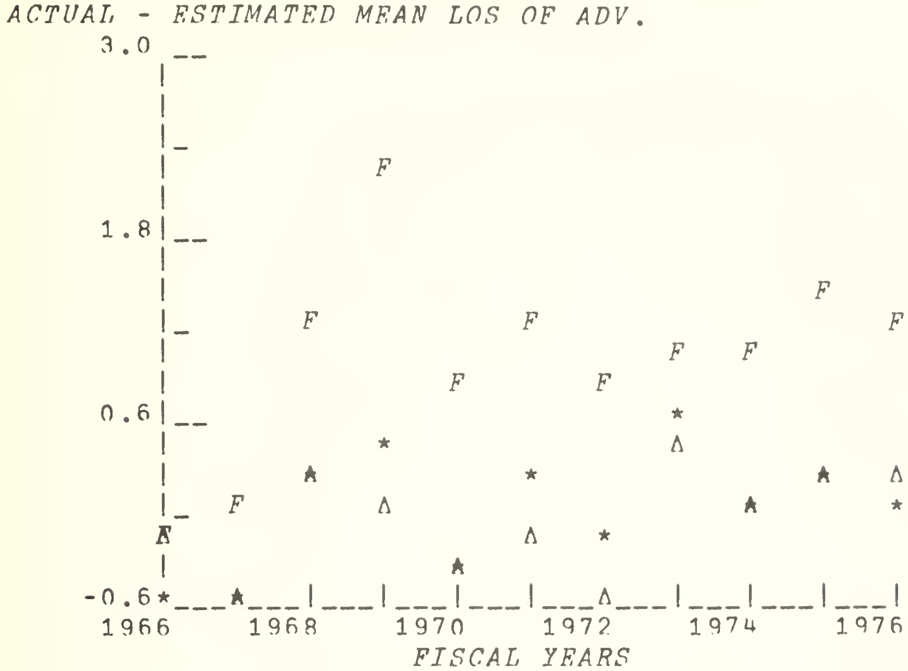
ACTUAL - ESTIMATED MEAN LOS OF ADV.

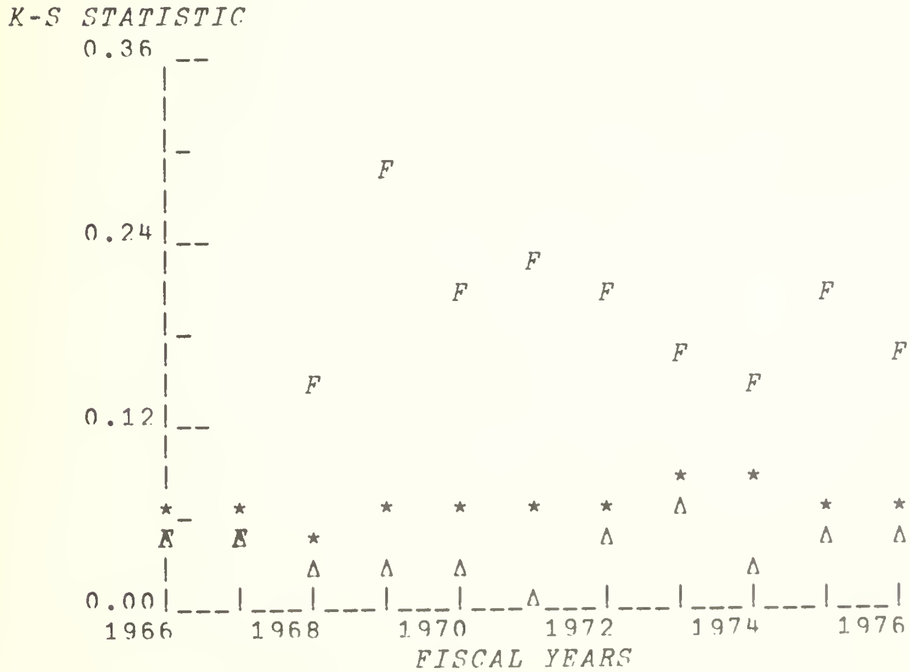


K-S STATISTIC

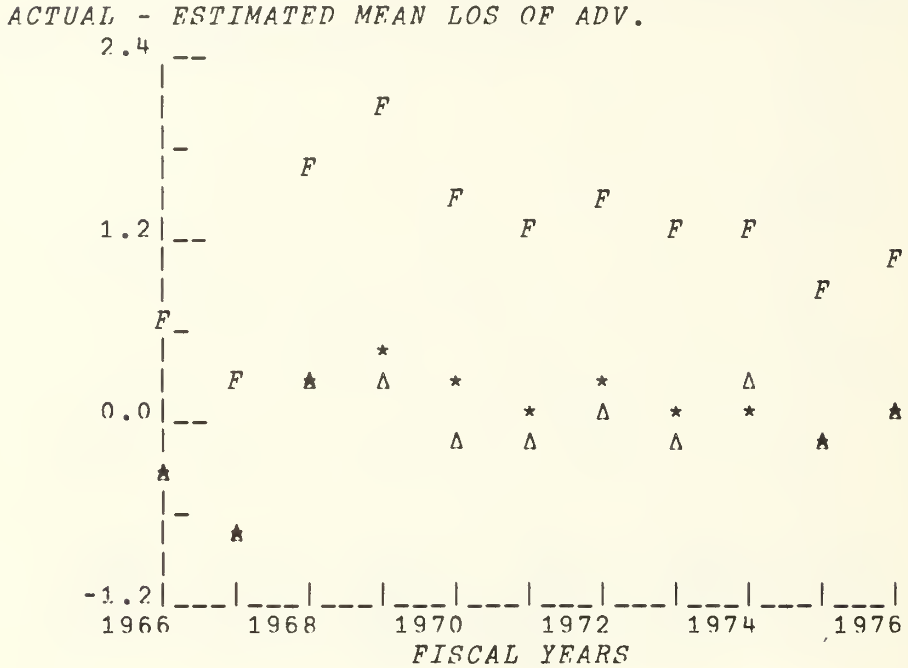


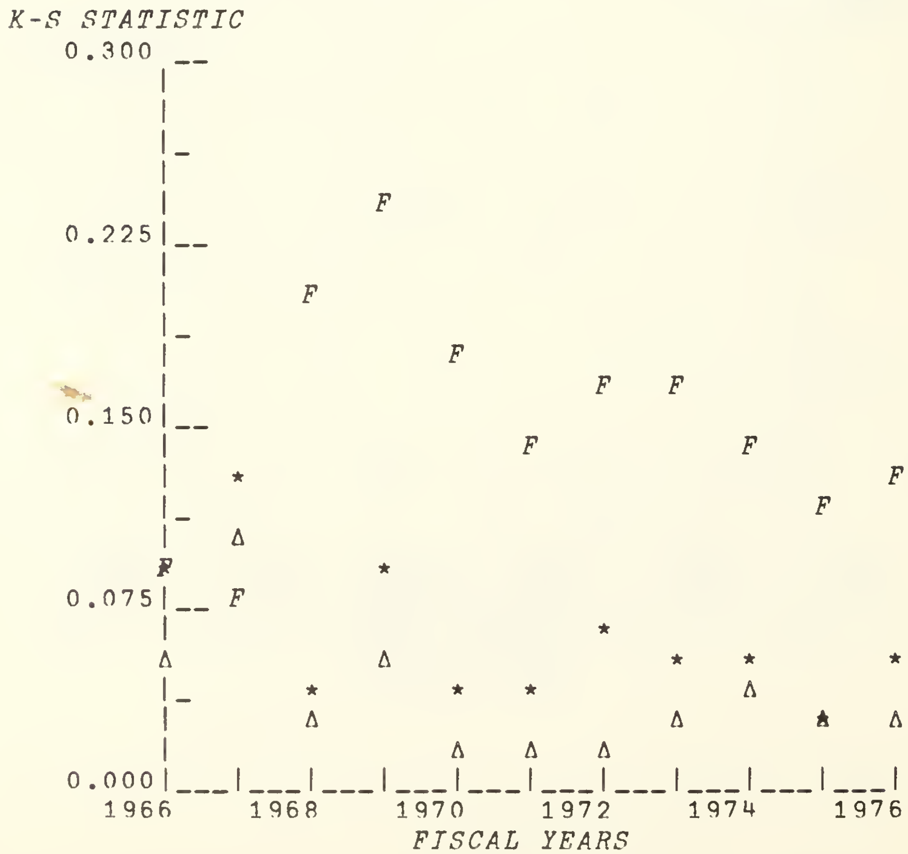
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=0 PAY GRADE=6
Δ: REGRESSION ★: GAMMA DIST. F: F A S T



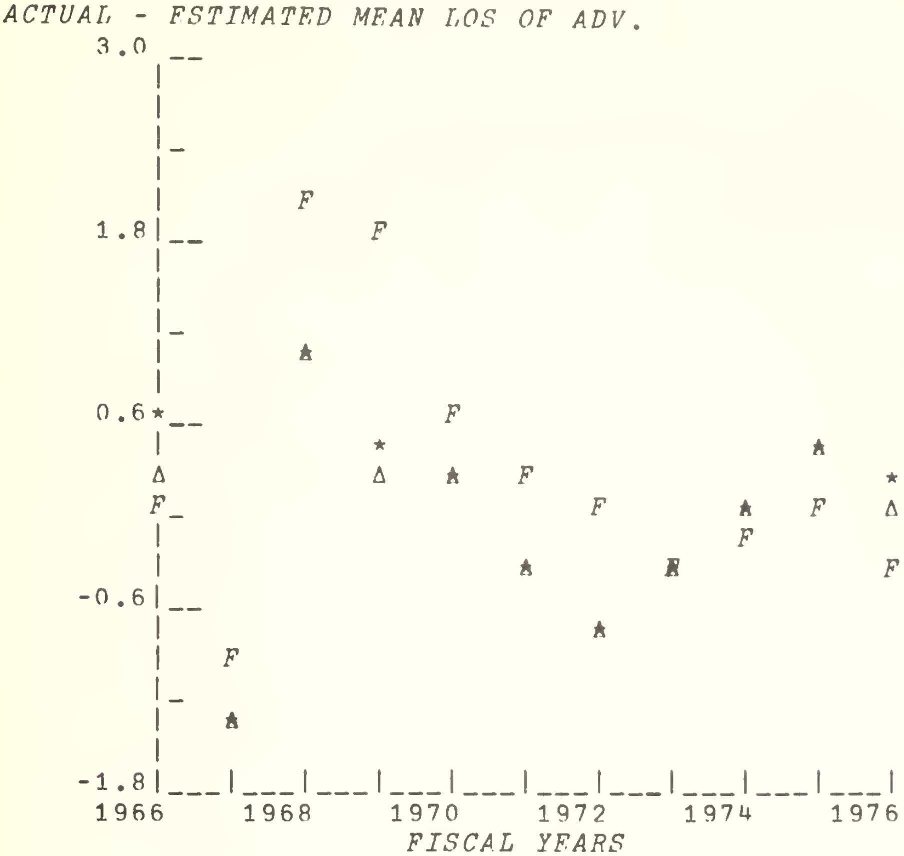


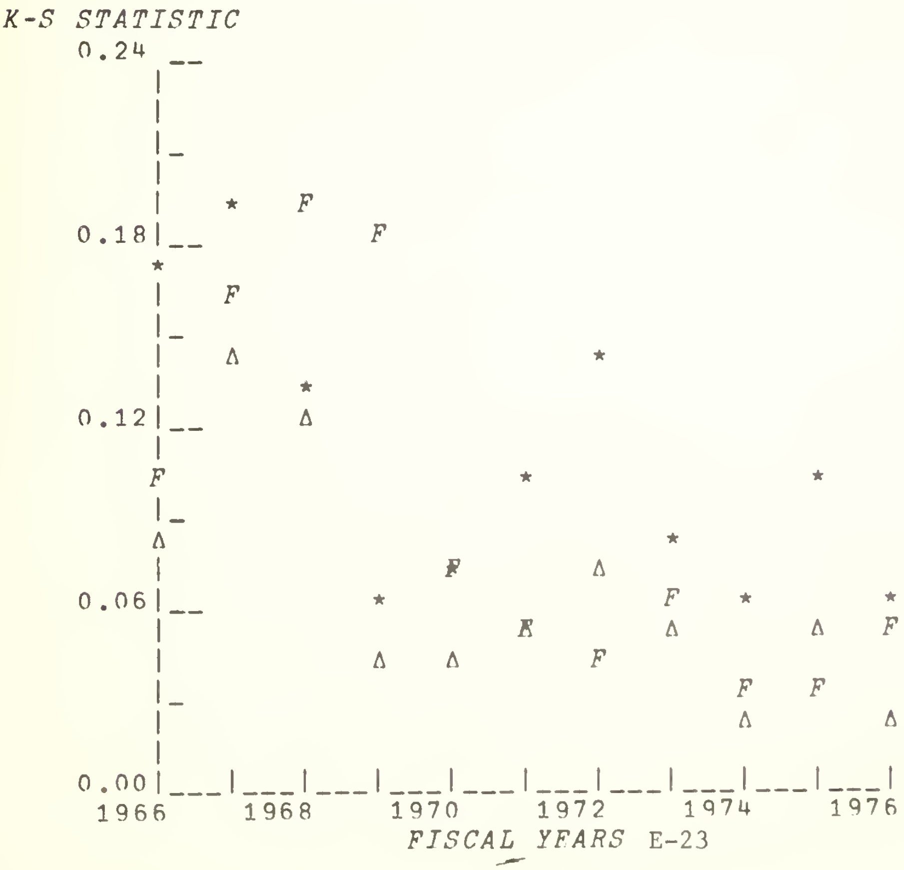
COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=0 PAY GRADE=7
*Δ: REGRESSION *: GAMMA DIST. F: F A S T*





COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS
RATING=0 PAY GRADE=8
Δ: REGRESSION *: GAMMA DIST. F: F A S T



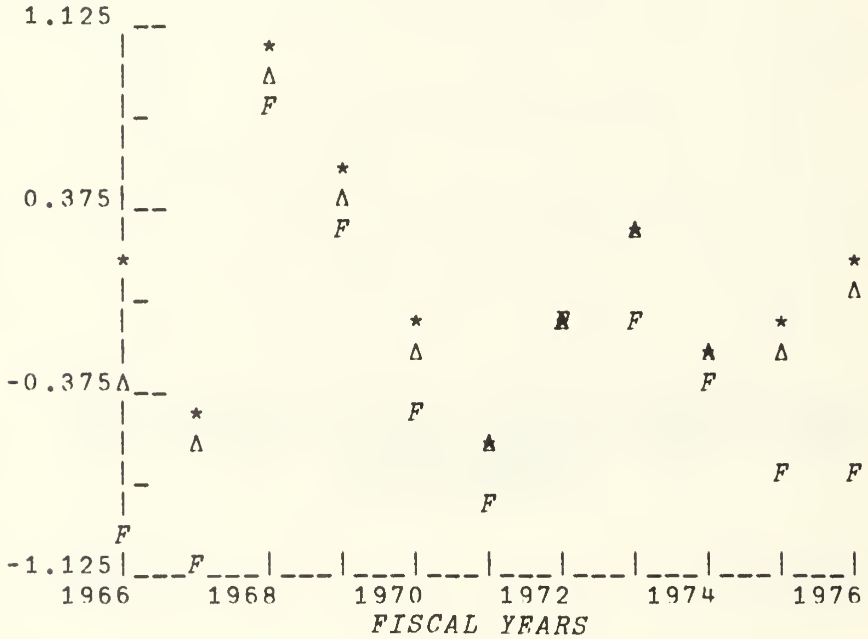


COMPARISON OF ERRORS OF ESTIMATED ADVANCEMENTS

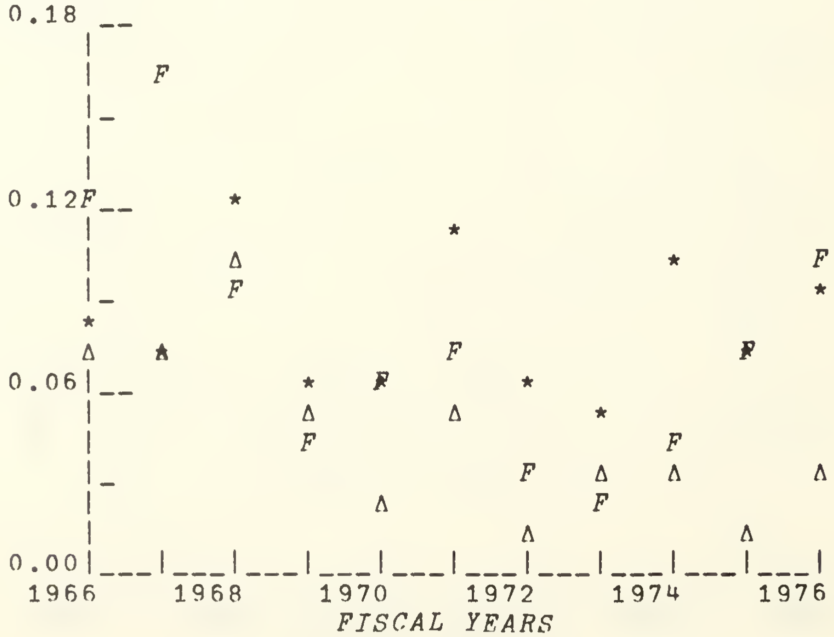
RATING=0 PAY GRADE=9

Δ: *REGRESSION* *: *GAMMA DIST.* *F: F A S T*

ACTUAL - ESTIMATED MEAN LOS OF ADV.



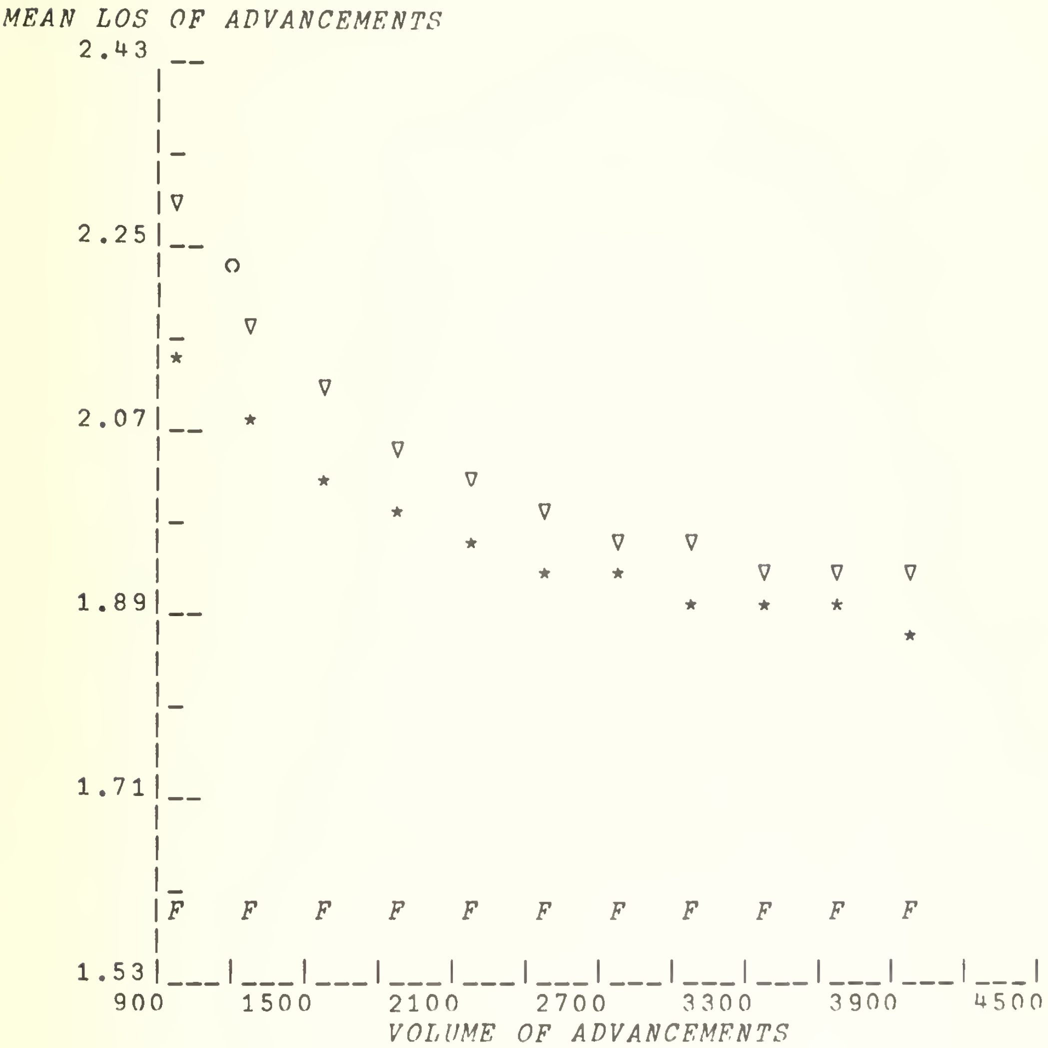
K-S STATISTIC



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=300 *PAY GRADE*=4 *YEAR*=1976

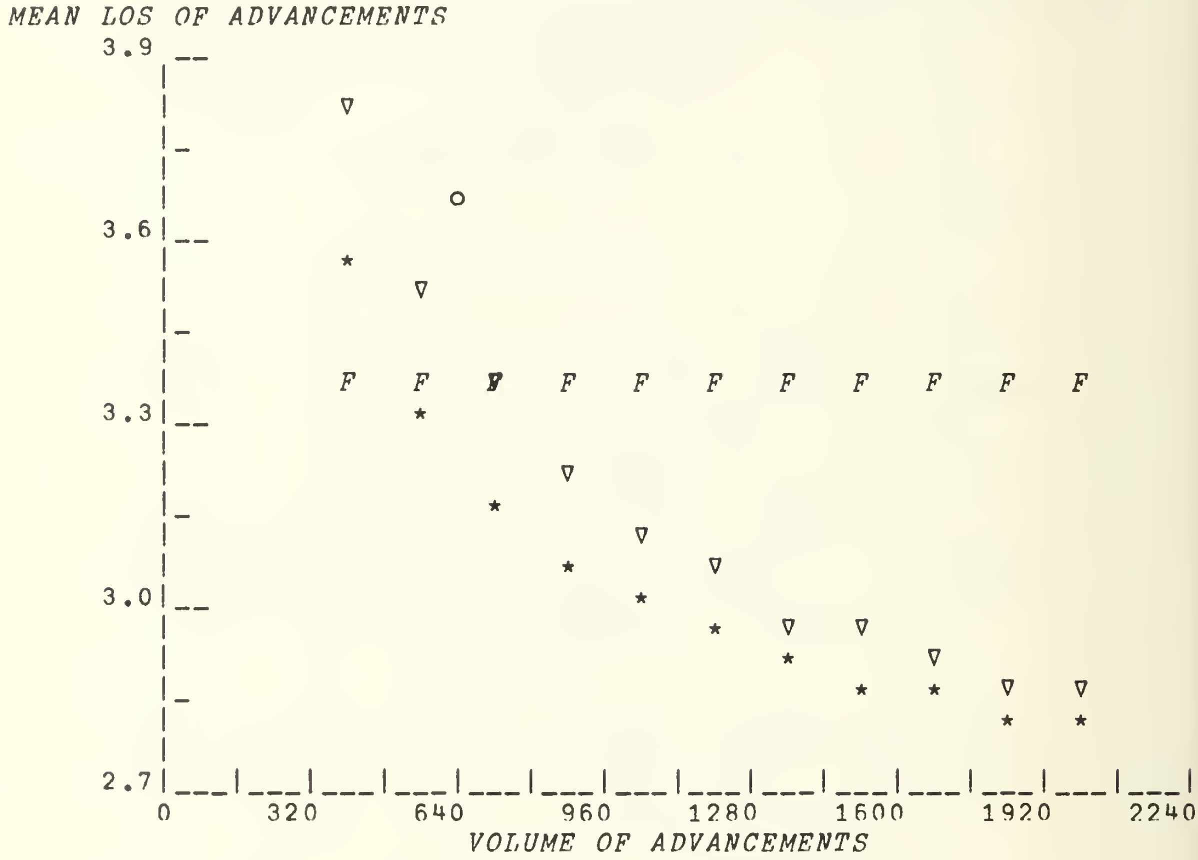
o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=300 *PAY GRADE*=5 *YEAR*=1976

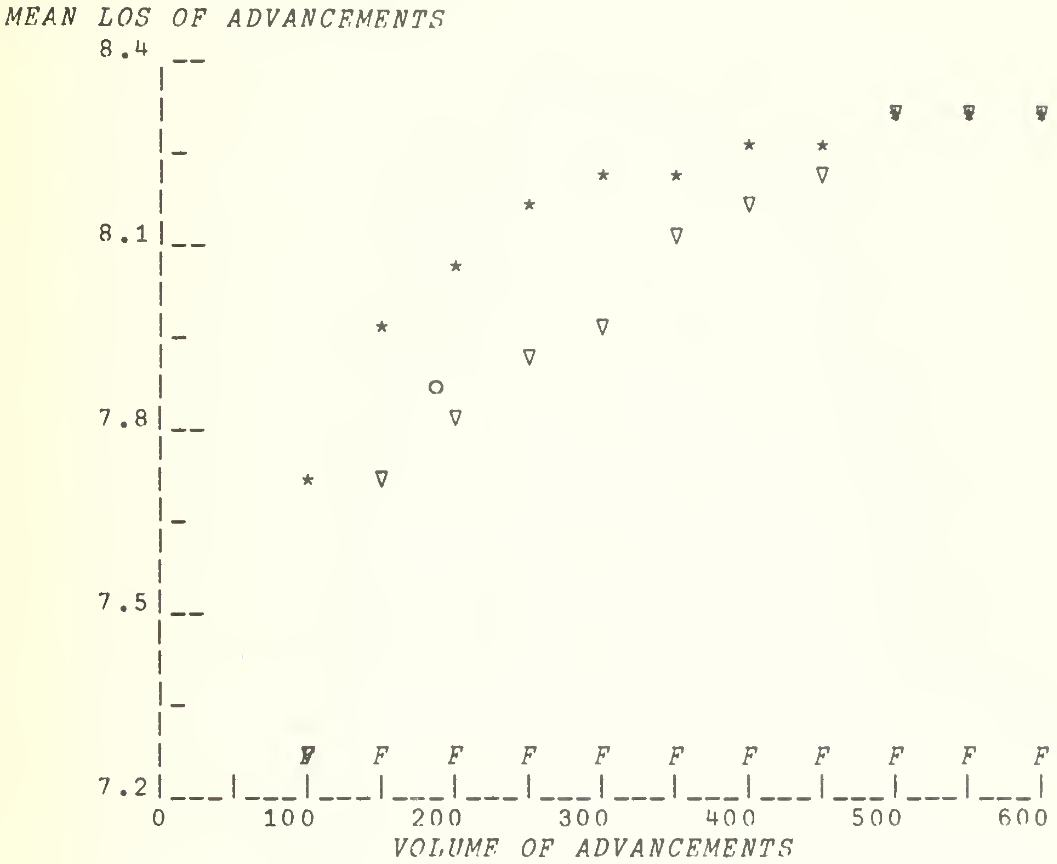
o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F: F A S T*



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=300 PAY GRADE=6 YEAR=1976

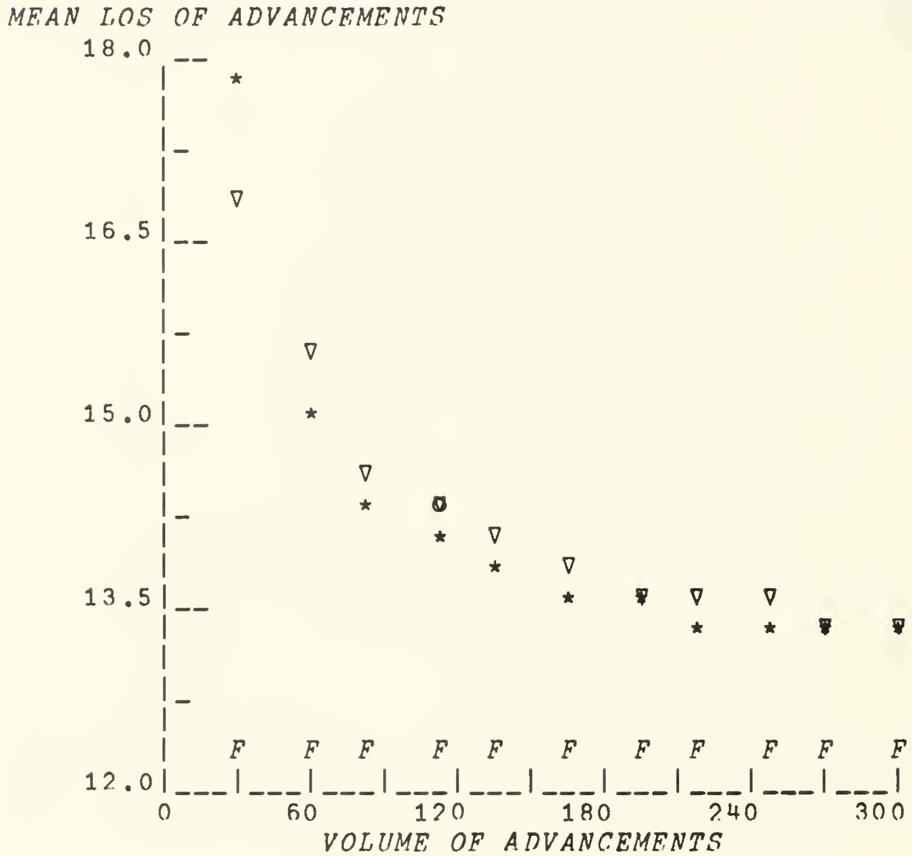
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T

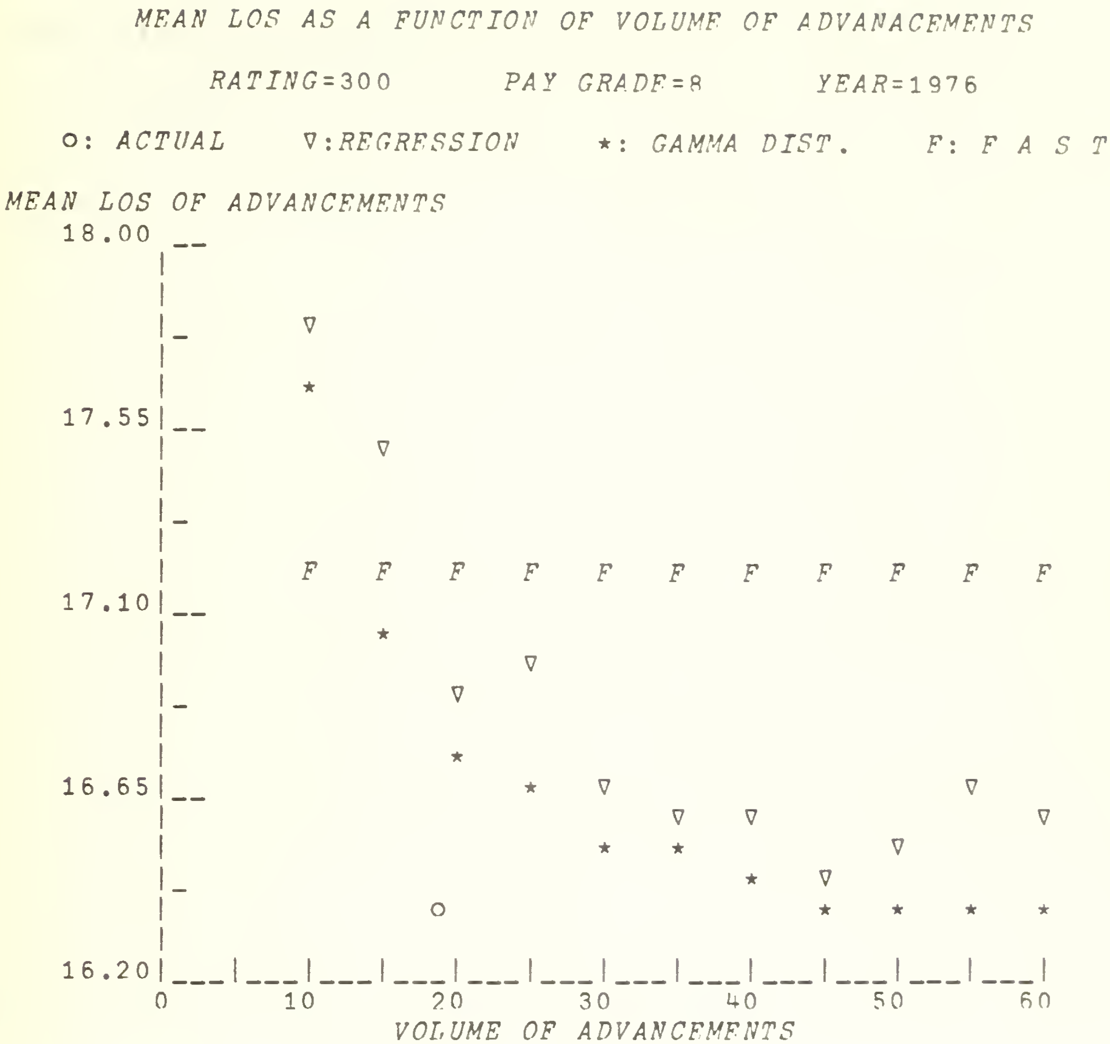


MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=300 *PAY GRADE*=7 *YEAR*=1976

o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*

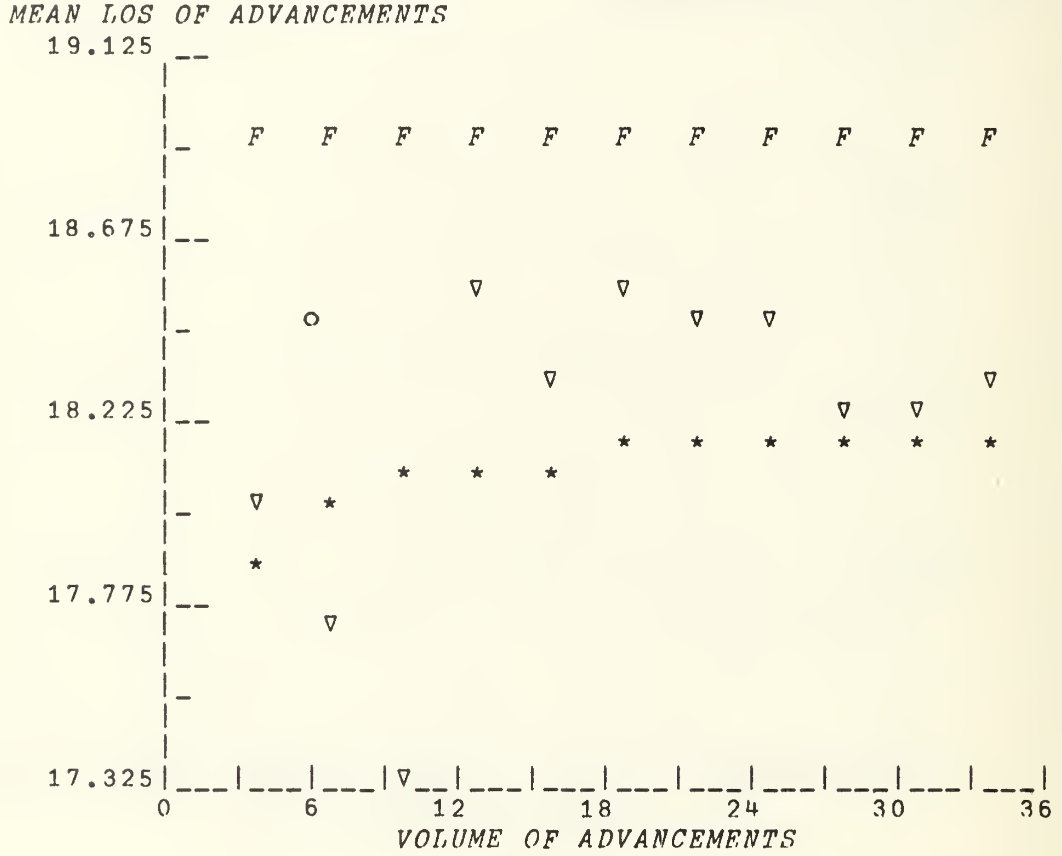


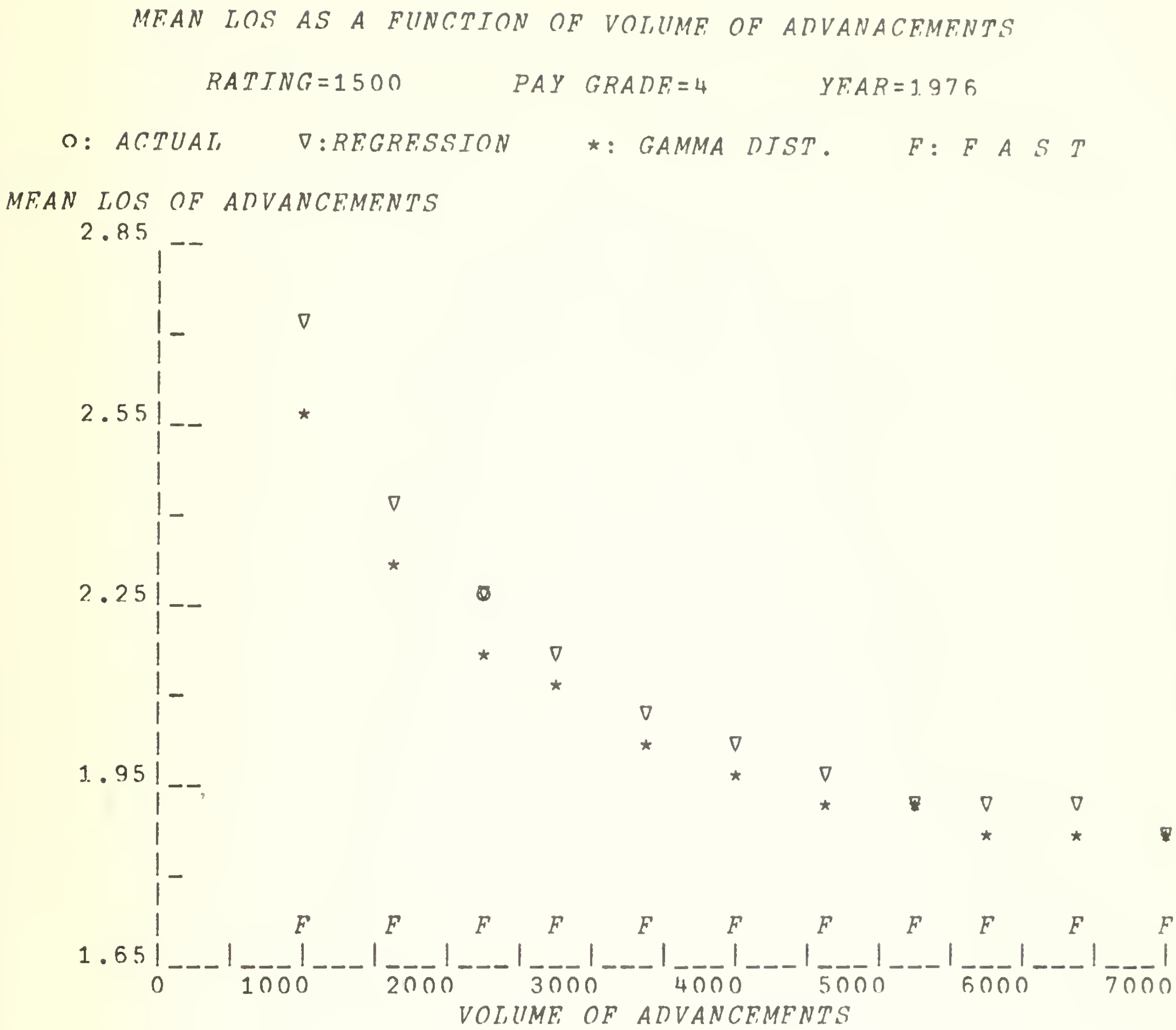

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=300
PAY GRADE=8
YEAR=1976
o: ACTUAL
∇:REGRESSION
*: GAMMA DIST.
F: F A S T
MEAN LOS OF ADVANCEMENTS
18.00
17.55
17.10
16.65
16.20
0
10
20
30
40
50
60
VOLUME OF ADVANCEMENTS

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=300 *PAY GRADE*=9 *YEAR*=1976

o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*

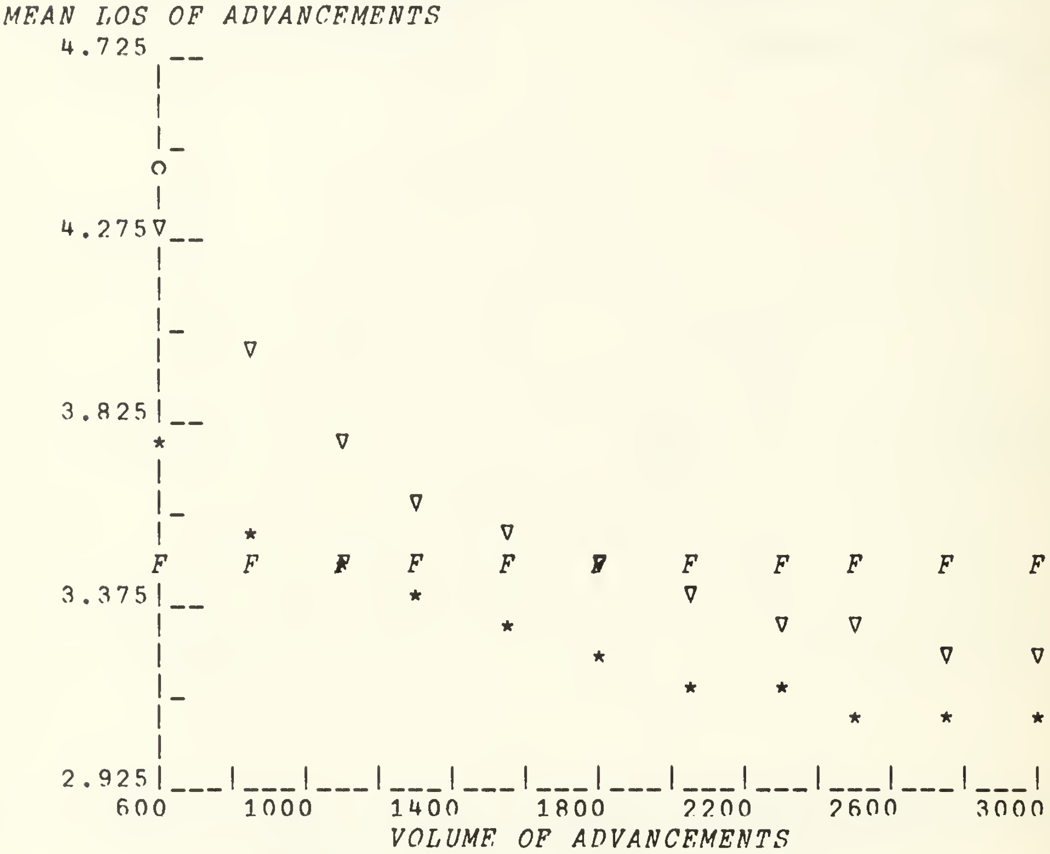



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=1500 PAY GRADE=4 YEAR=1976
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
2.85
2.55
2.25
1.95
1.65
0
1000
2000
3000
4000
5000
6000
7000
VOLUME OF ADVANCEMENTS

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=1500 *PAY GRADE*=5 *YEAR*=1976

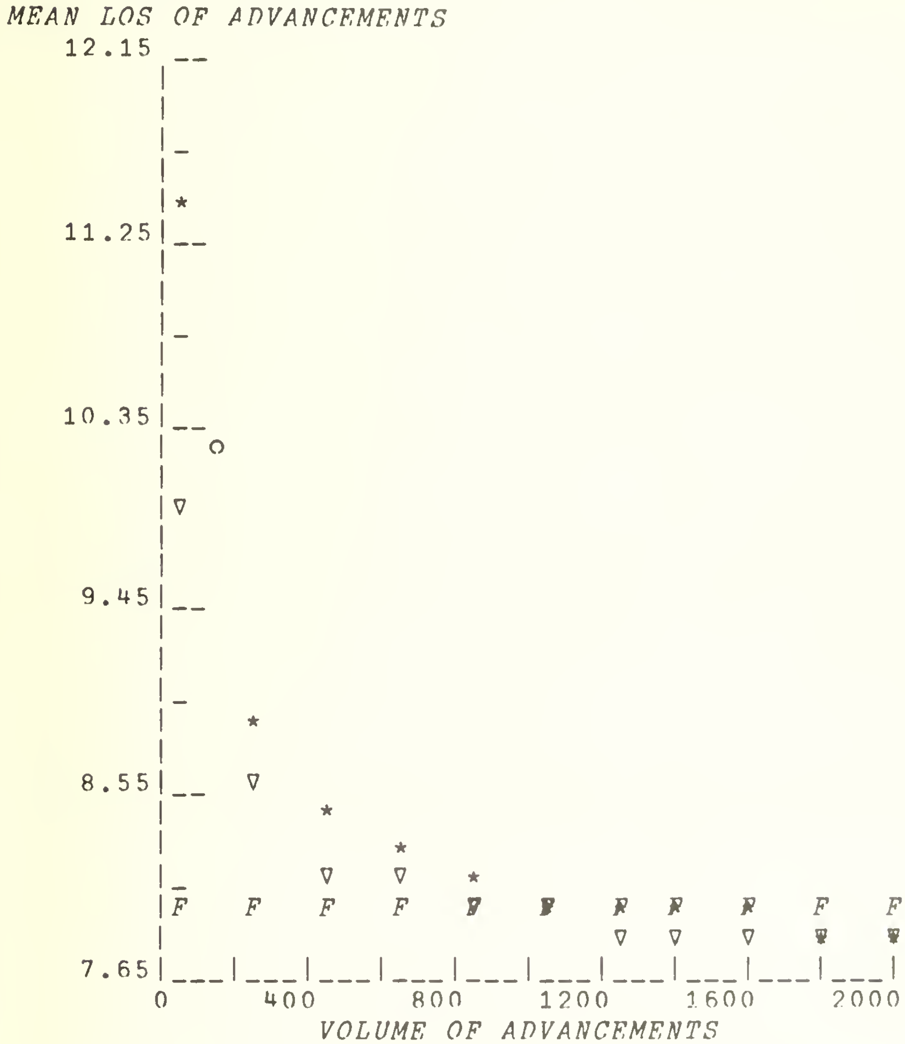
o: *ACTUAL* ∇:*REGRESSION* ⋆: *GAMMA DIST.* *F*: *F A S T*



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=1500 *PAY GRADE*=6 *YEAR*=1976

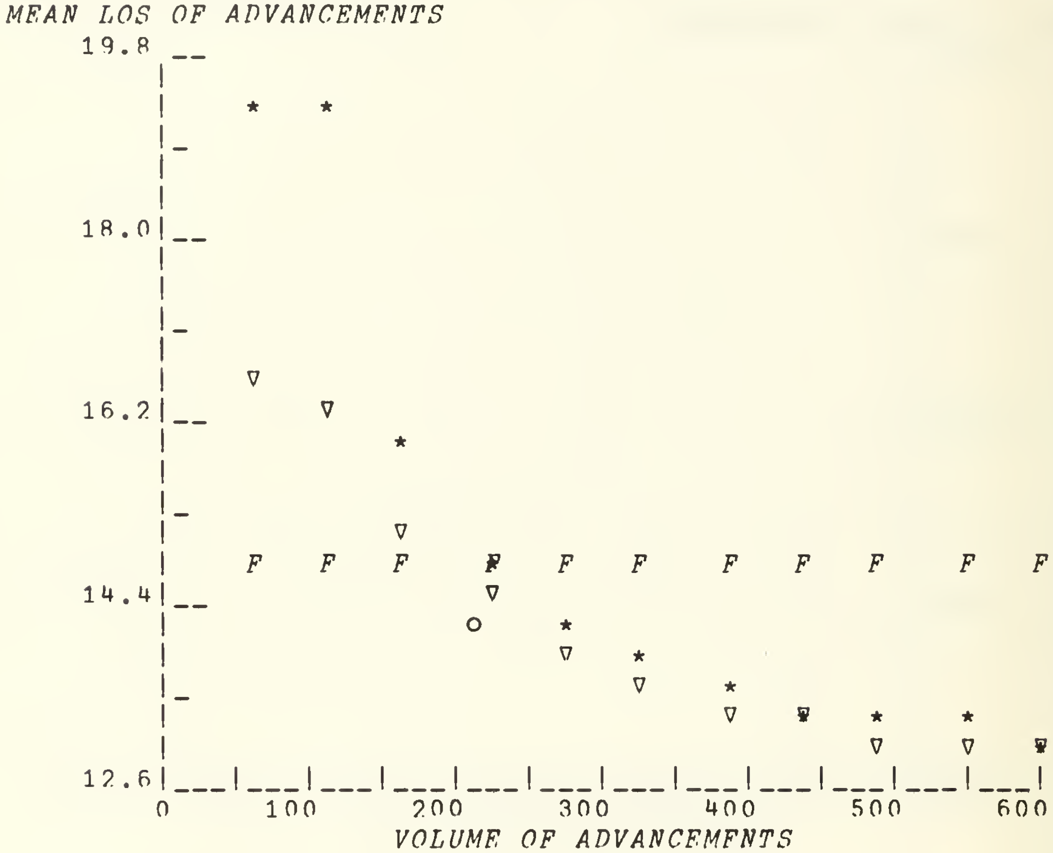
o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*

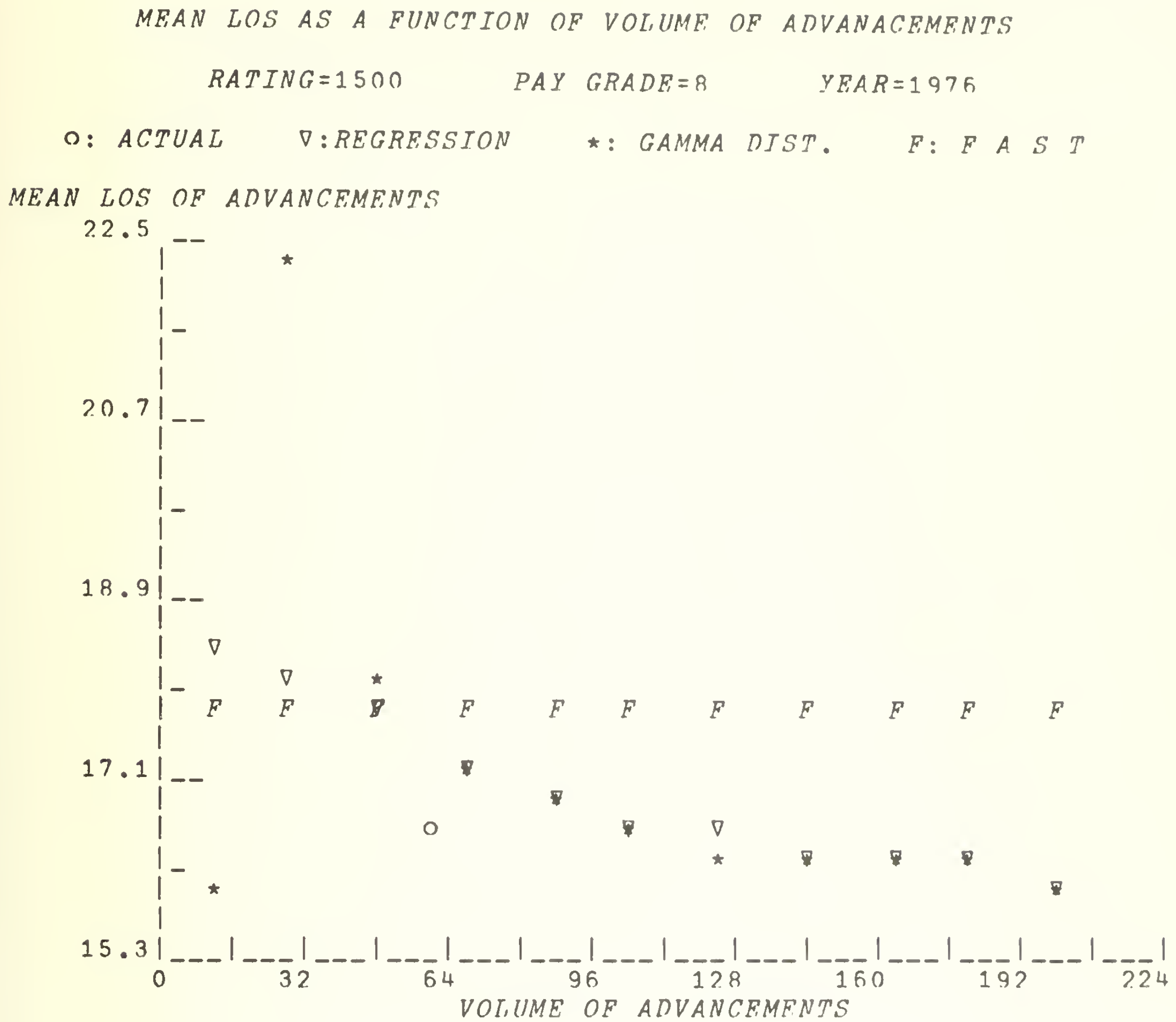


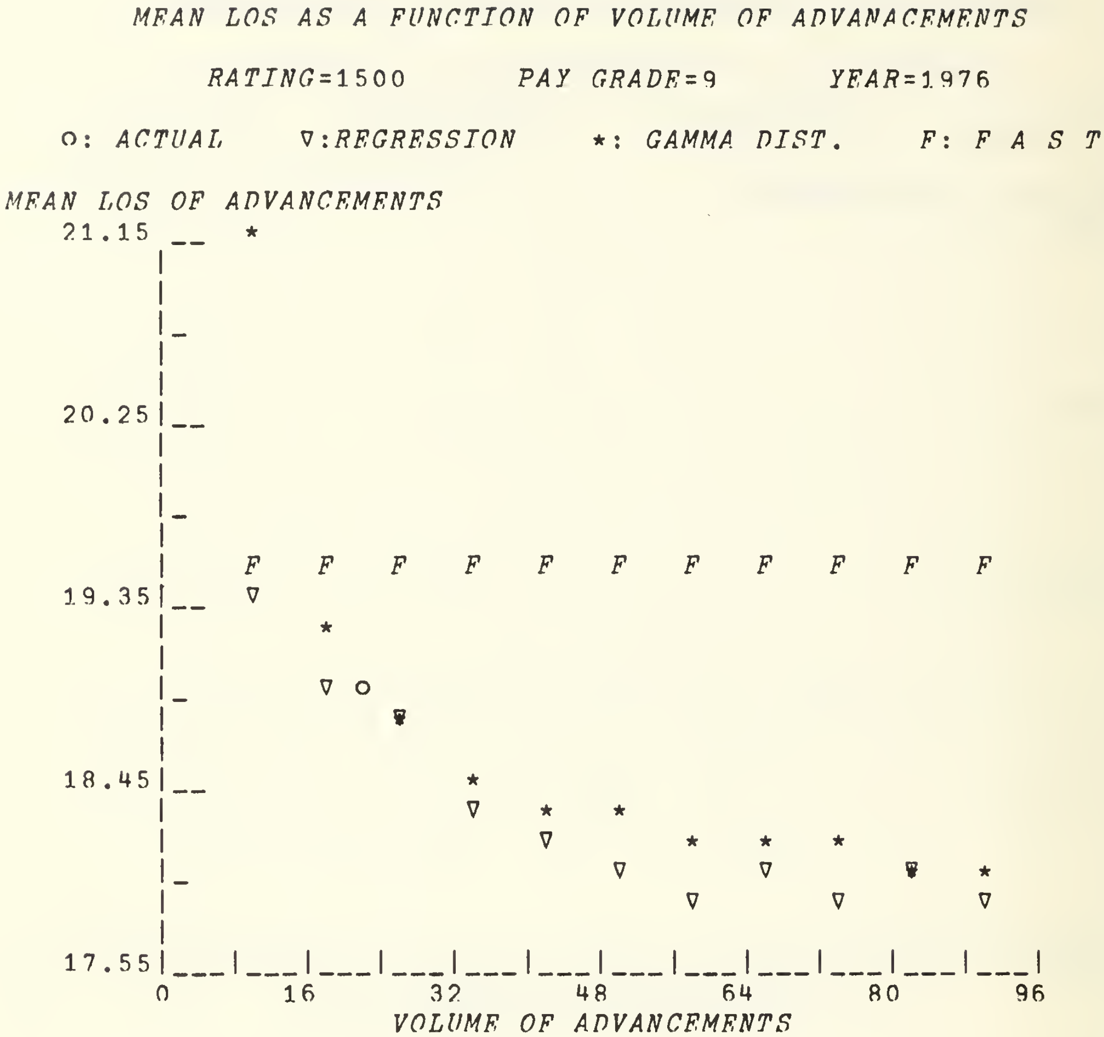
MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

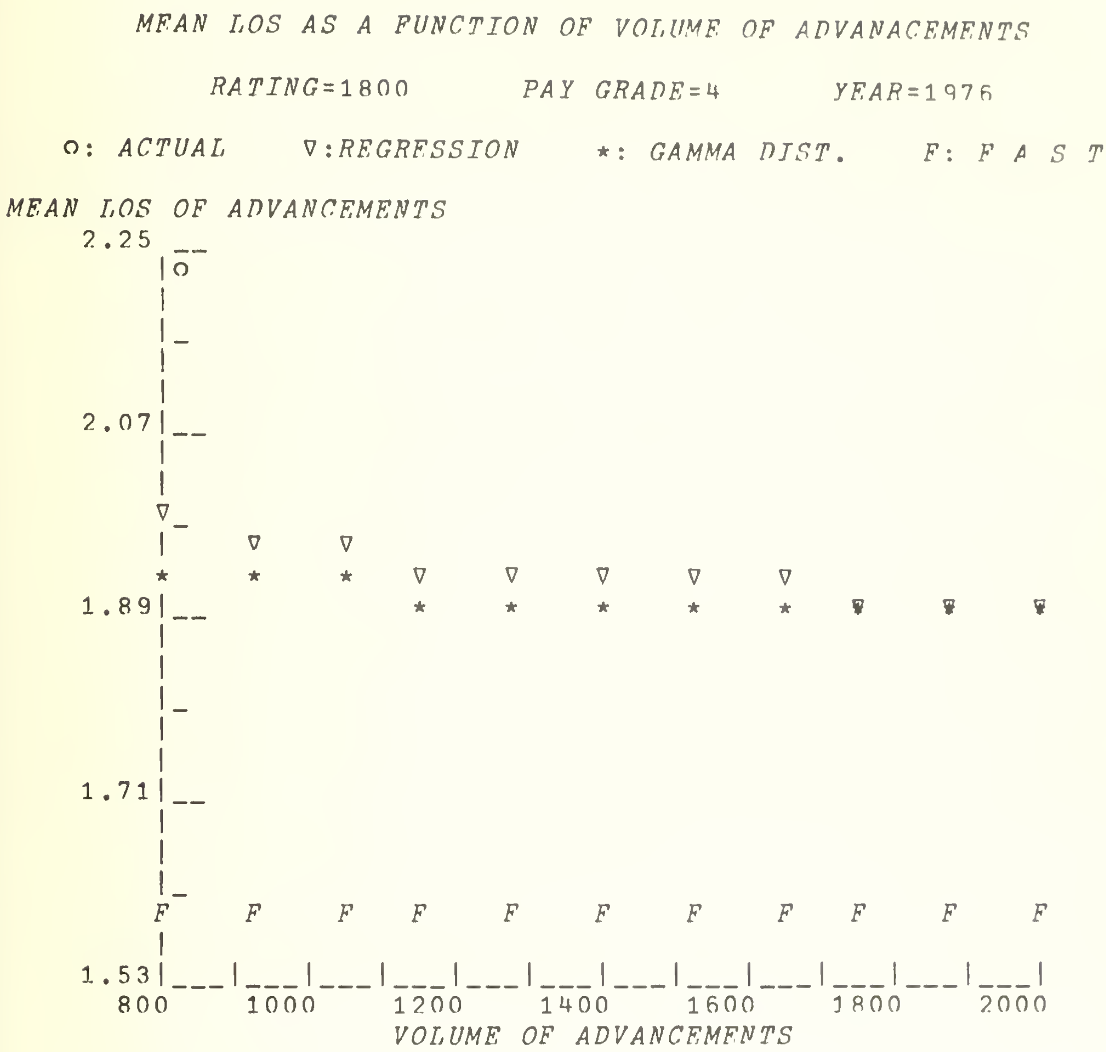
RATING=1500 *PAY GRADE*=7 *YEAR*=1976

o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* F: *F A S T*




MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=1500 PAY GRADE=8 YEAR=1976
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
22.5
20.7
18.9
17.1
15.3
0
32
64
96
128
160
192
224
VOLUME OF ADVANCEMENTS

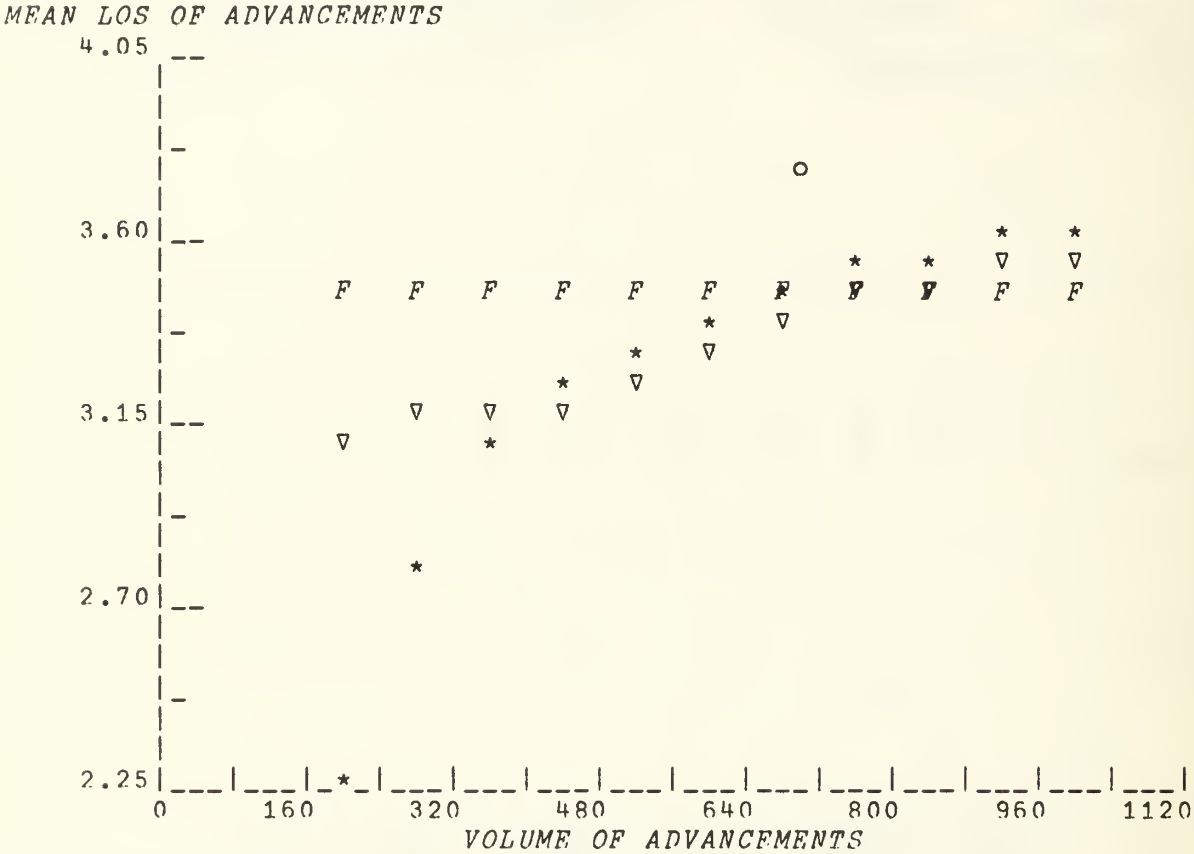

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=1500 PAY GRADE=9 YEAR=1976
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
21.15
20.25
19.35
18.45
17.55
0
16
32
48
64
80
96
VOLUME OF ADVANCEMENTS


MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=1800 PAY GRADE=4 YEAR=1976
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
2.25
2.07
1.89
1.71
1.53
800
1000
1200
1400
1600
1800
2000
VOLUME OF ADVANCEMENTS

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=1800 PAY GRADE=5 YEAR=1976

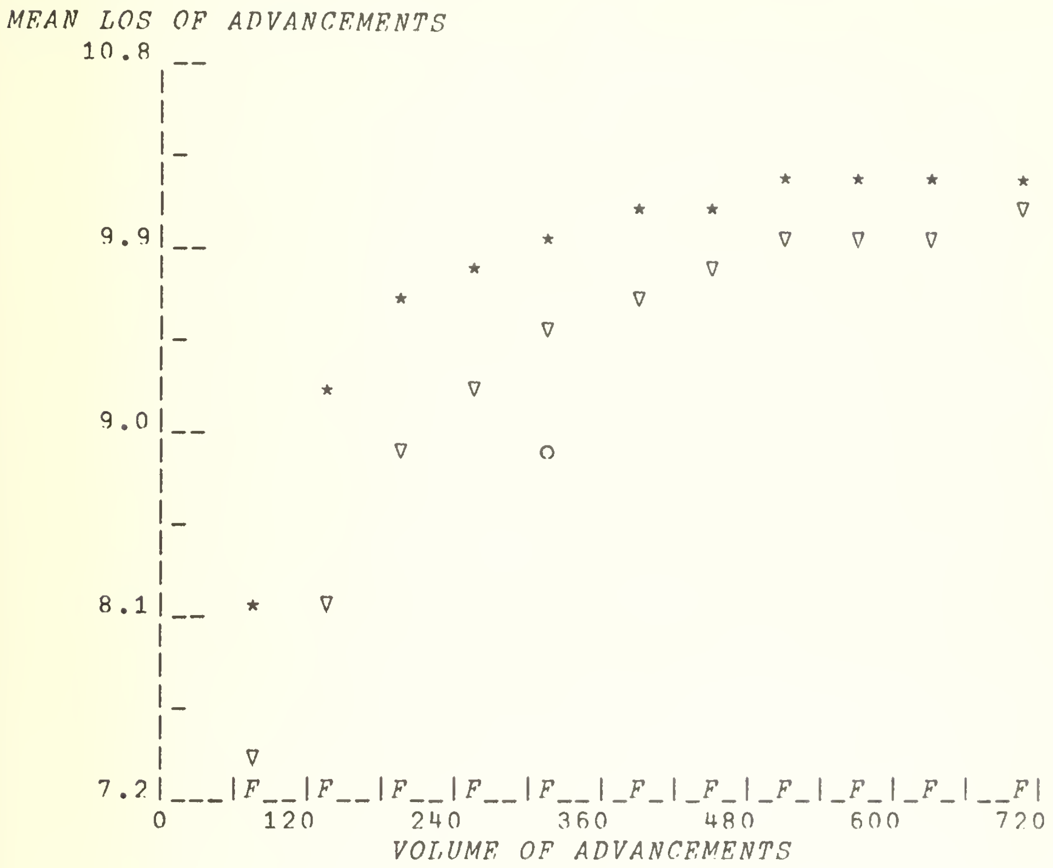
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T

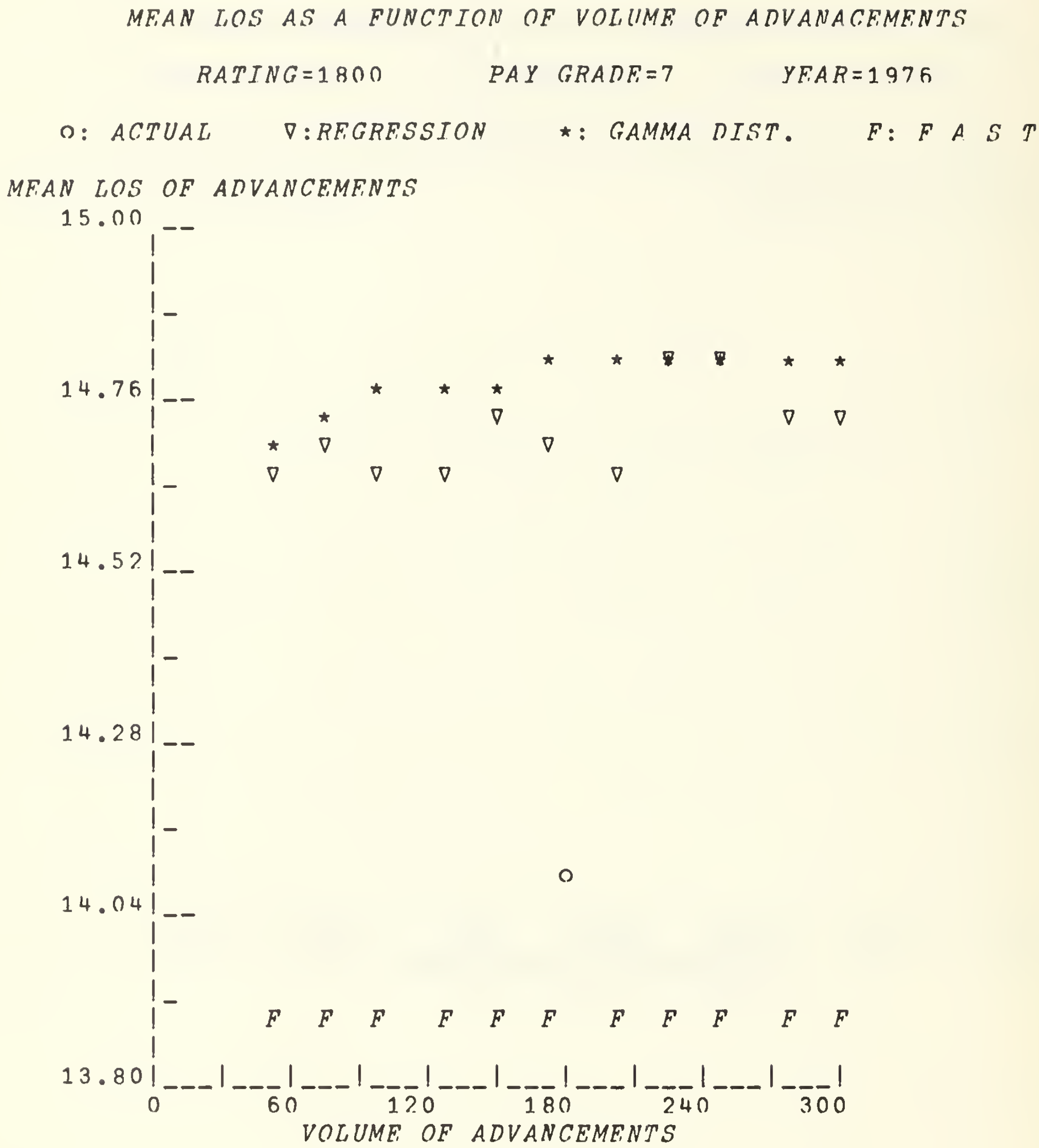


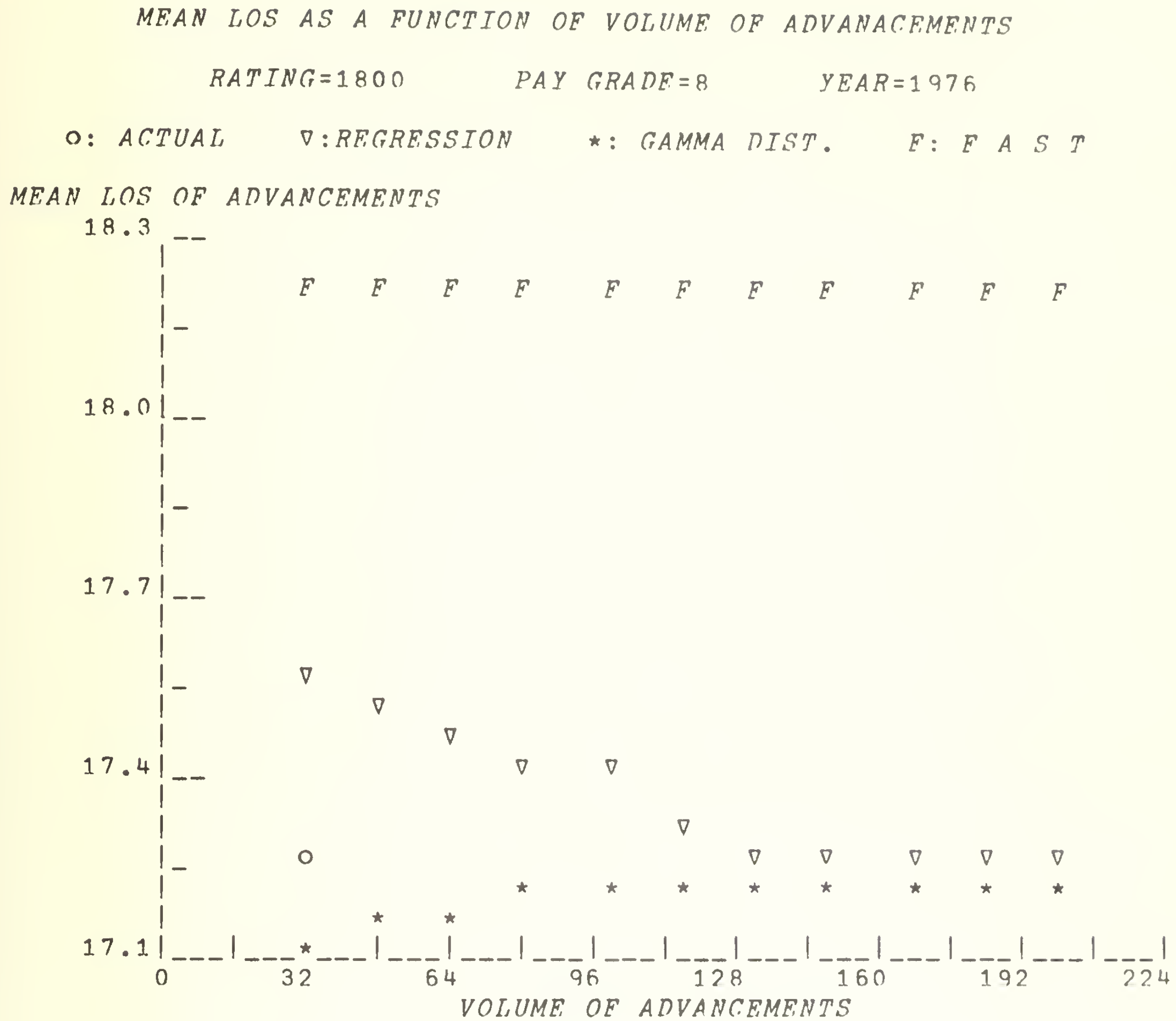
MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=1800 PAY GRADE=6 YEAR=1976

o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T



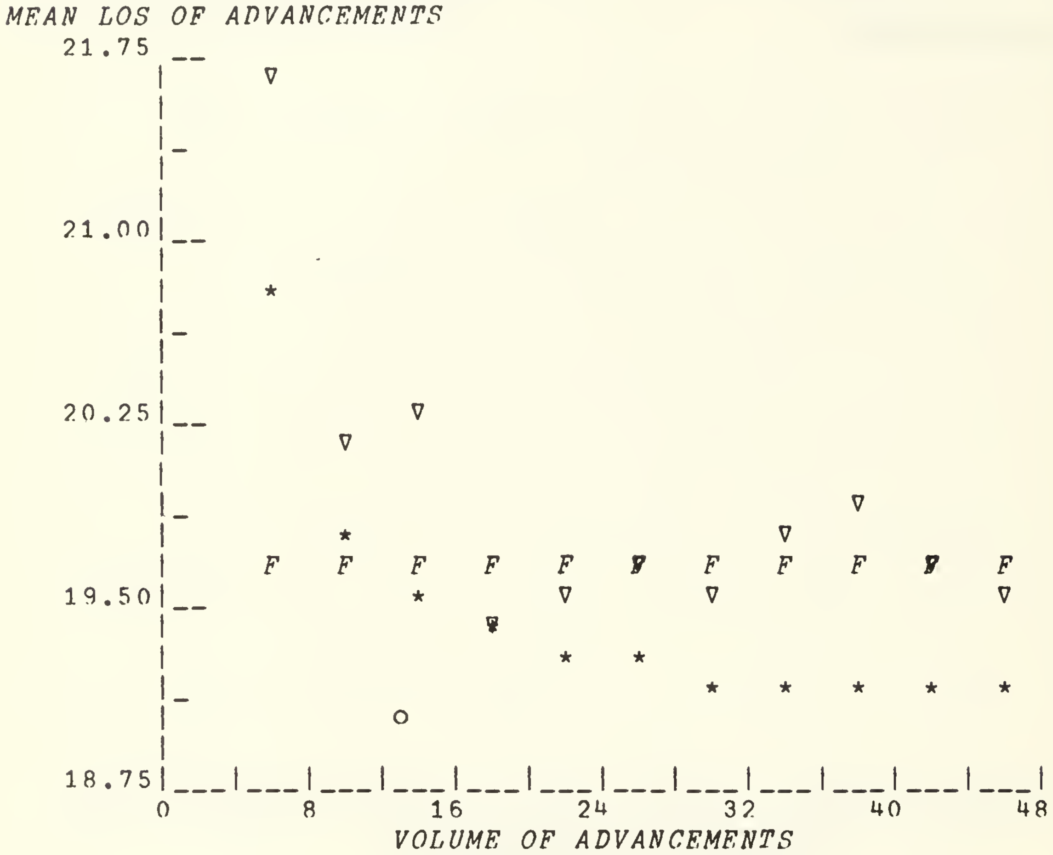

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=1800 PAY GRADE=7 YEAR=1976
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
15.00
14.76
14.52
14.28
14.04
13.80
0
60
120
180
240
300
VOLUME OF ADVANCEMENTS

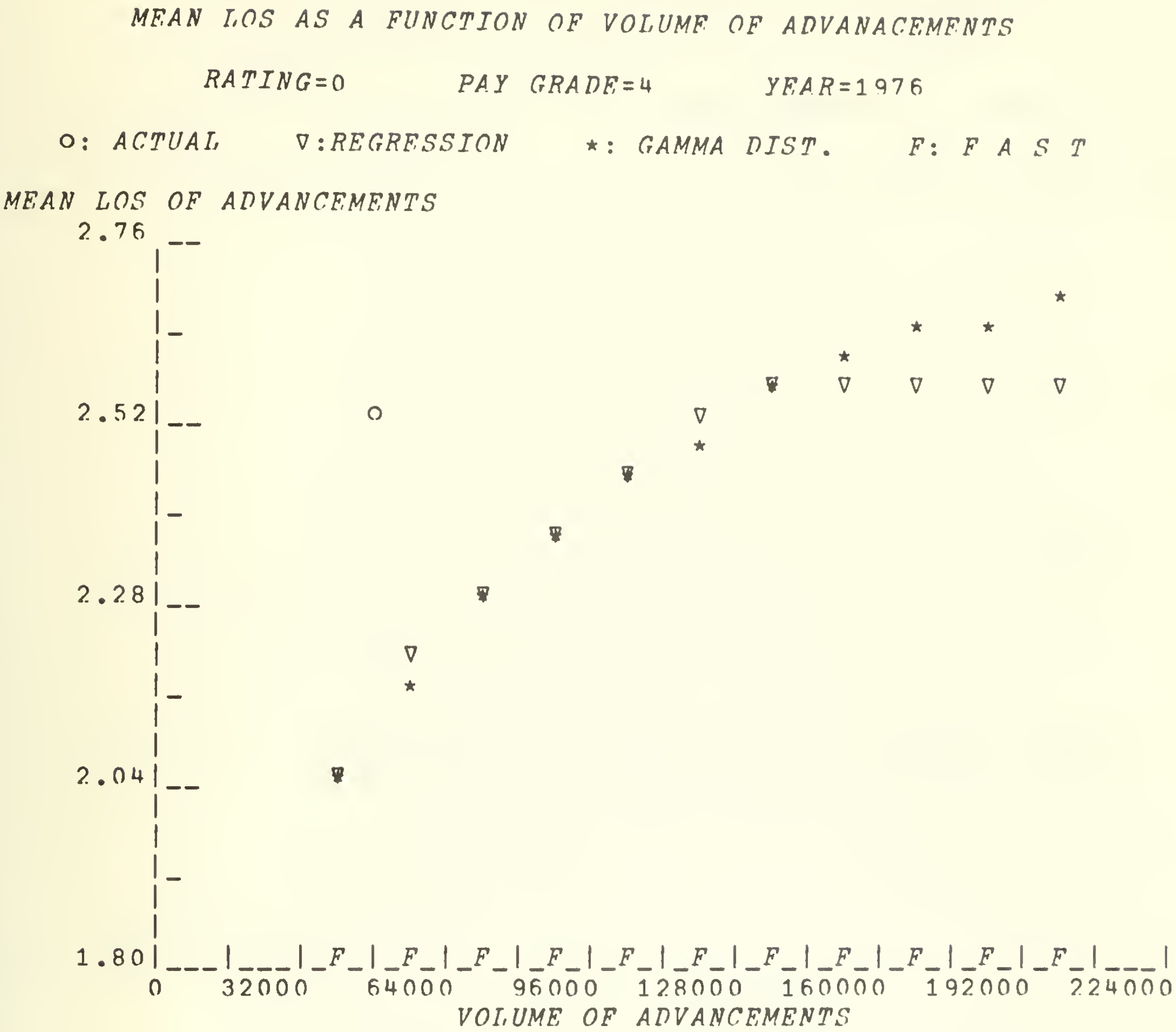

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=1800 PAY GRADE=8 YEAR=1976
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
18.3
18.0
17.7
17.4
17.1
0
32
64
96
128
160
192
224
VOLUME OF ADVANCEMENTS

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=1800 *PAY GRADE*=9 *YEAR*=1976

o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*

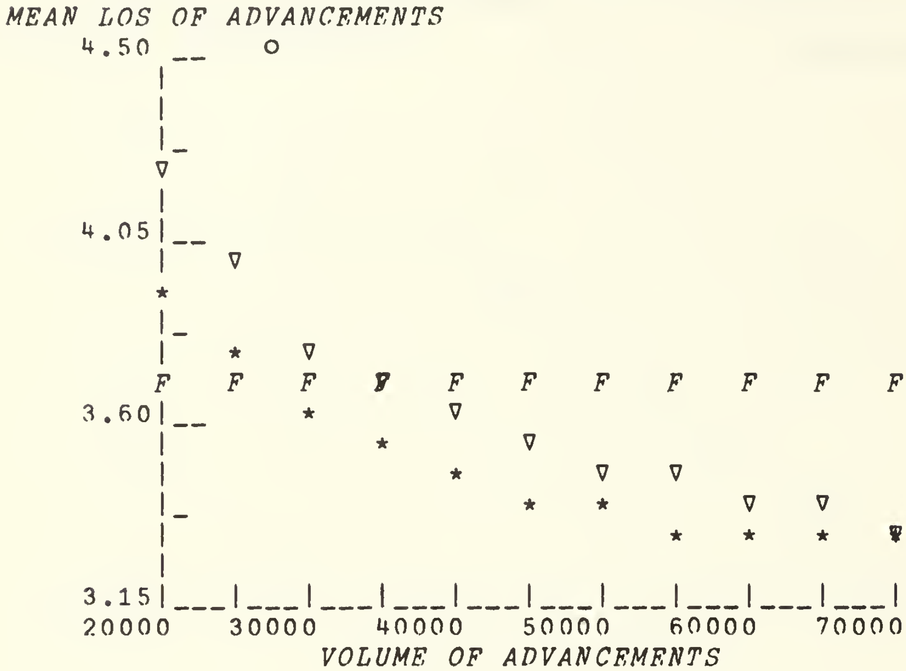



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=0 PAY GRADE=4 YEAR=1976
O: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
2.76
2.52
2.28
2.04
1.80
0
32000
64000
96000
128000
160000
192000
224000
VOLUME OF ADVANCEMENTS

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=0 *PAY GRADE*=5 *YEAR*=1976

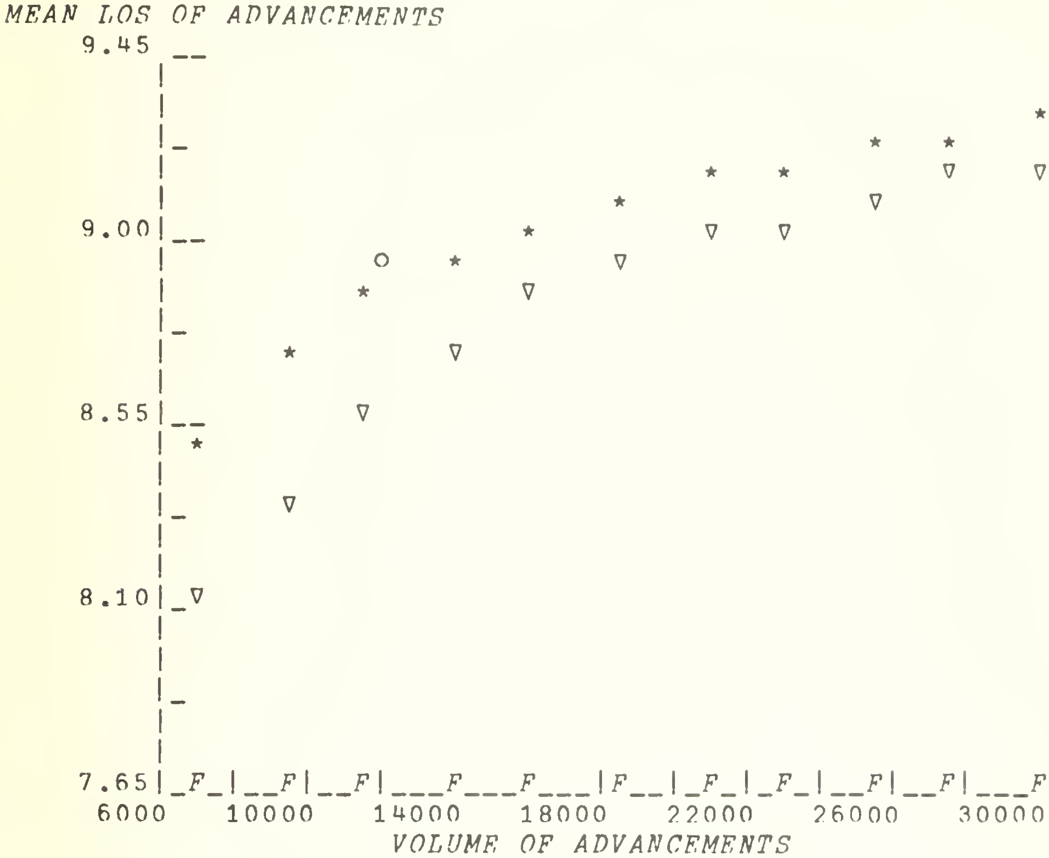
o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=0   PAY GRADE=6   YEAR=1976

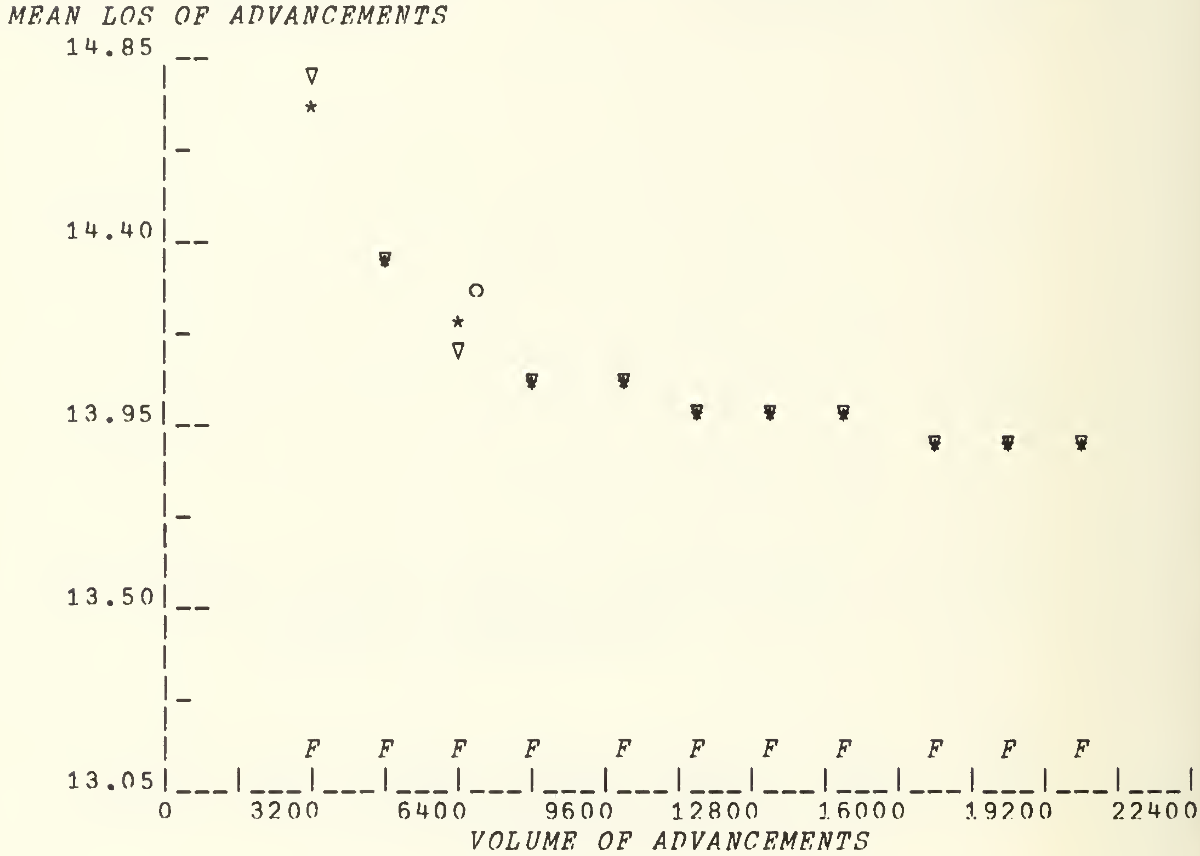
o: *ACTUAL*   ∇:*REGRESSION*   *: *GAMMA DIST.*   *F: F A S T*

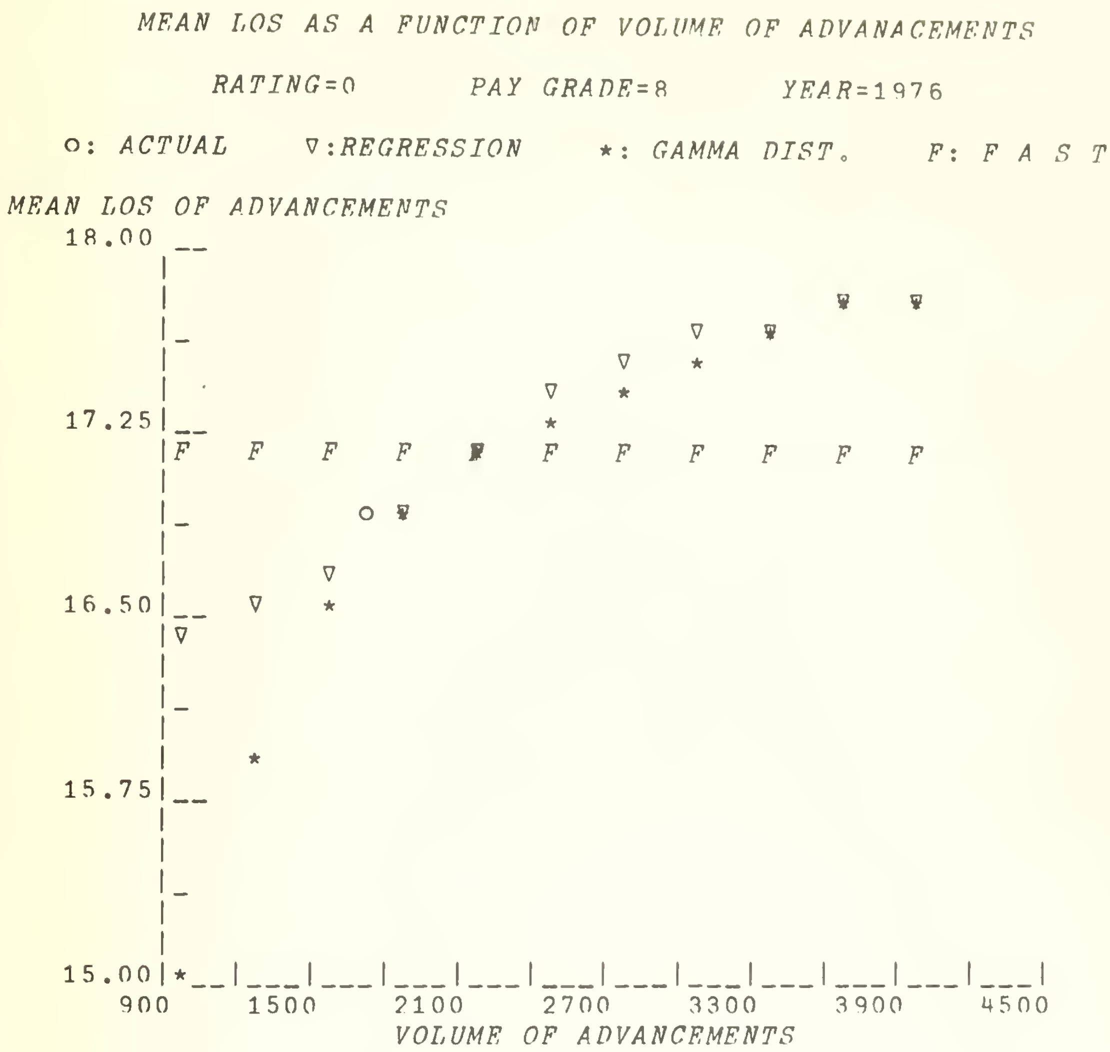


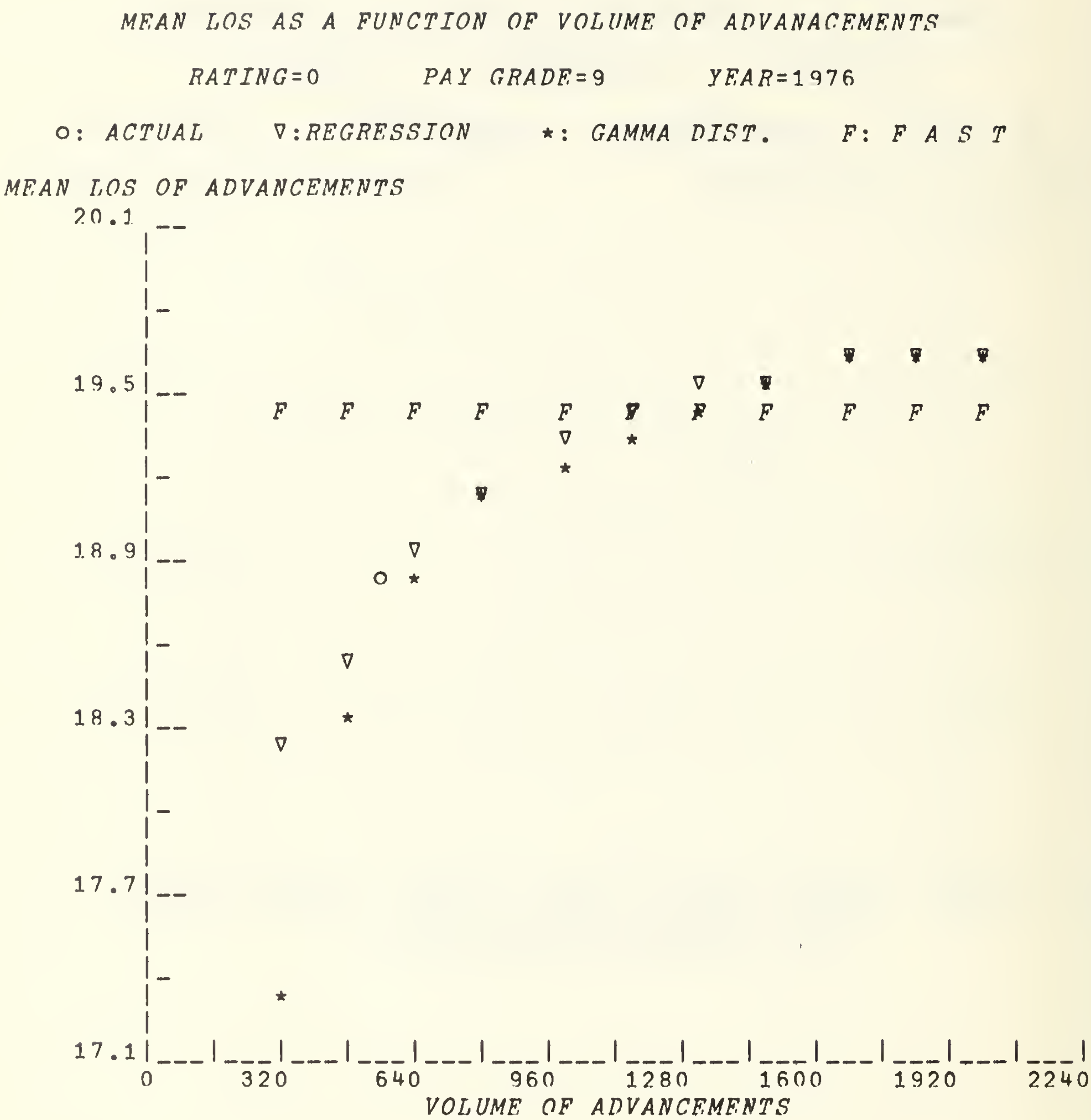
MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS

RATING=0 *PAY GRADE*=7 *YEAR*=1976

o: *ACTUAL* ∇:*REGRESSION* *: *GAMMA DIST.* *F*: *F A S T*

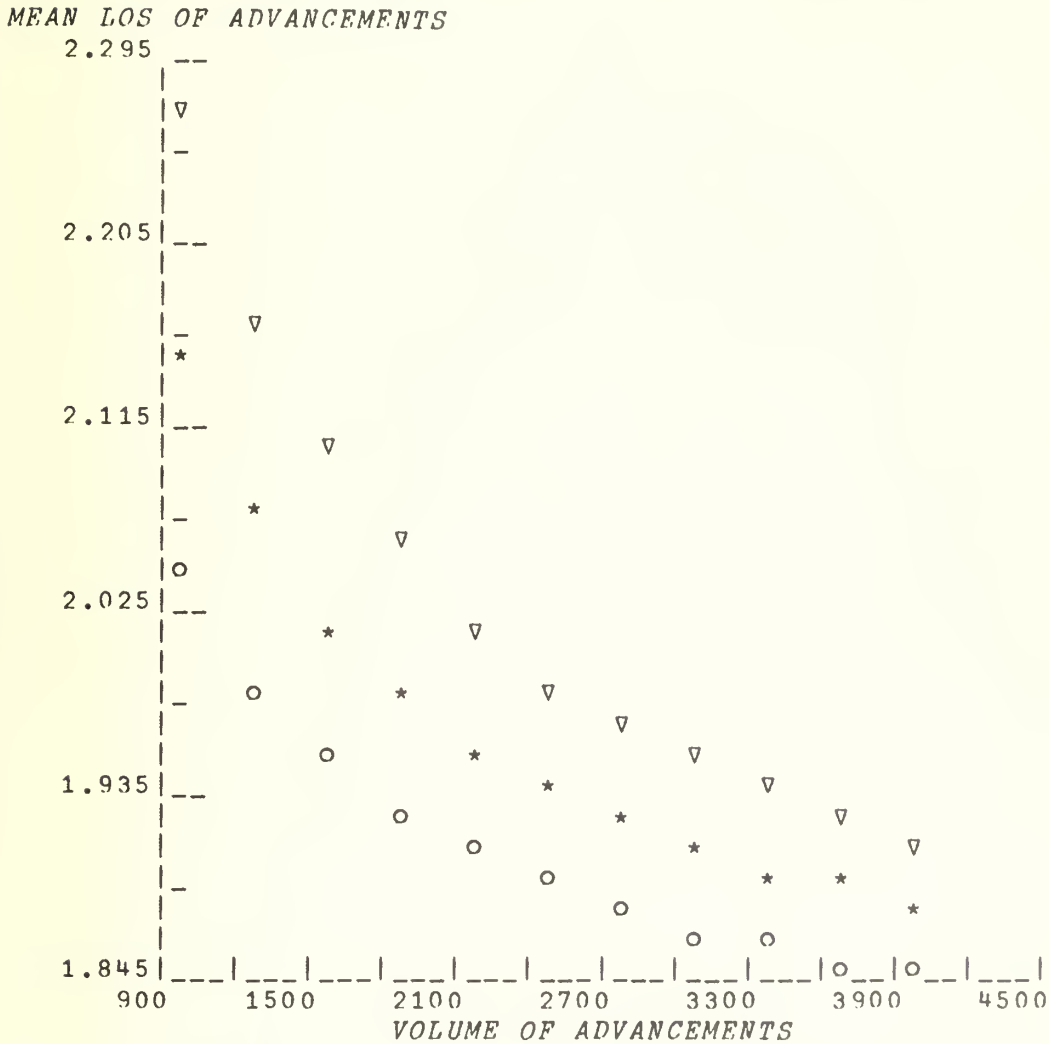



MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=0 PAY GRADE=8 YEAR=1976
o: ACTUAL ∇:REGRESSION *: GAMMA DIST. F: F A S T
MEAN LOS OF ADVANCEMENTS
18.00
17.25
16.50
15.75
15.00
900
1500
2100
2700
3300
3900
4500
VOLUME OF ADVANCEMENTS


MEAN LOS AS A FUNCTION OF VOLUME OF ADVANACEMENTS
RATING=0
PAY GRADE=9
YEAR=1976
o: ACTUAL
∇:REGRESSION
*: GAMMA DIST.
F: F A S T
MEAN LOS OF ADVANCEMENTS
20.1
19.5
18.9
18.3
17.7
17.1
0
320
640
960
1280
1600
1920
2240
VOLUME OF ADVANCEMENTS

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=300 *PAY GRADE=4*

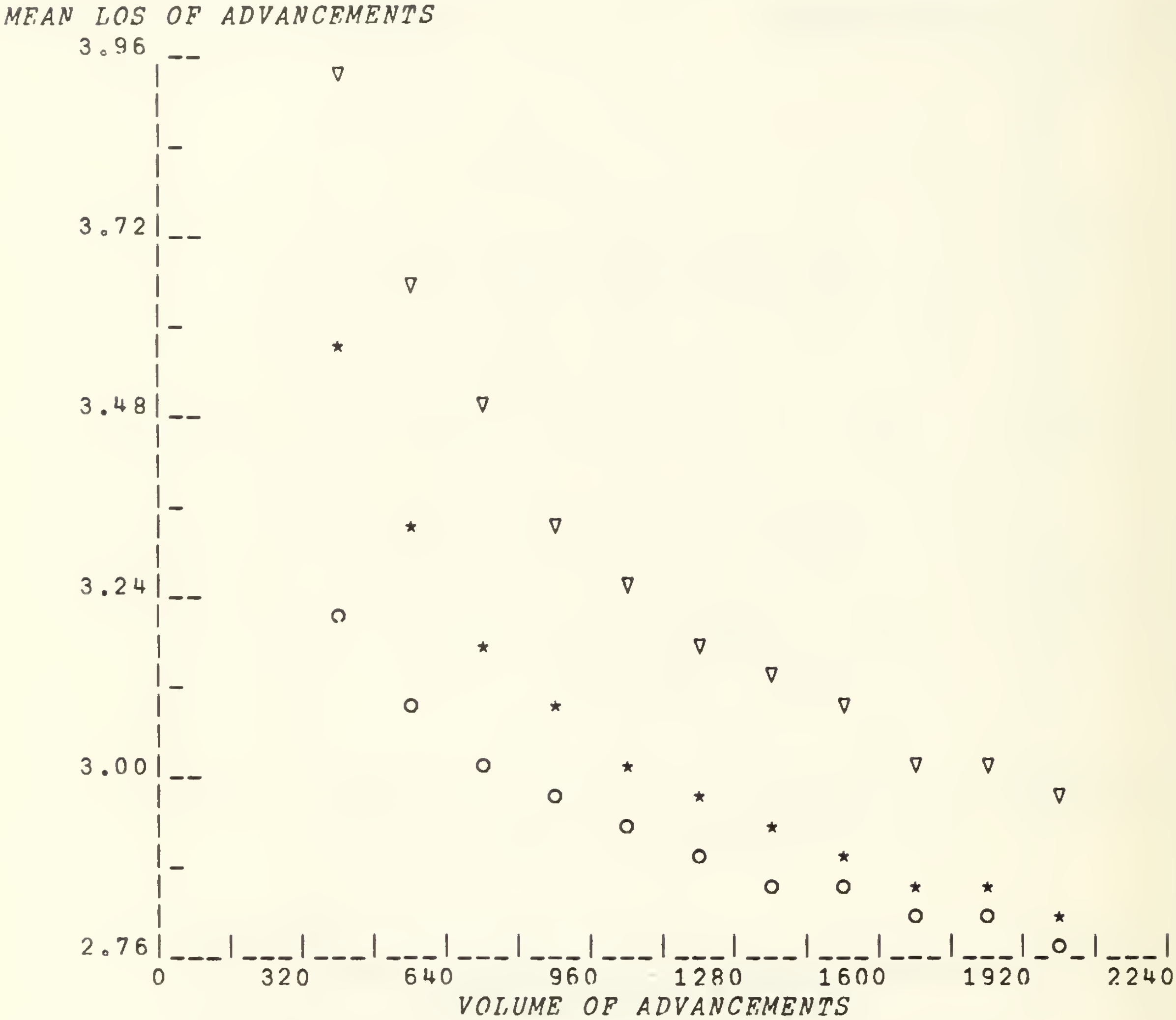


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	1204	2.24	1.60
o 1974	1257	1.94	1.70
∇ 1969	3858	1.89	1.54

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=300 *PAY GRADE=5*

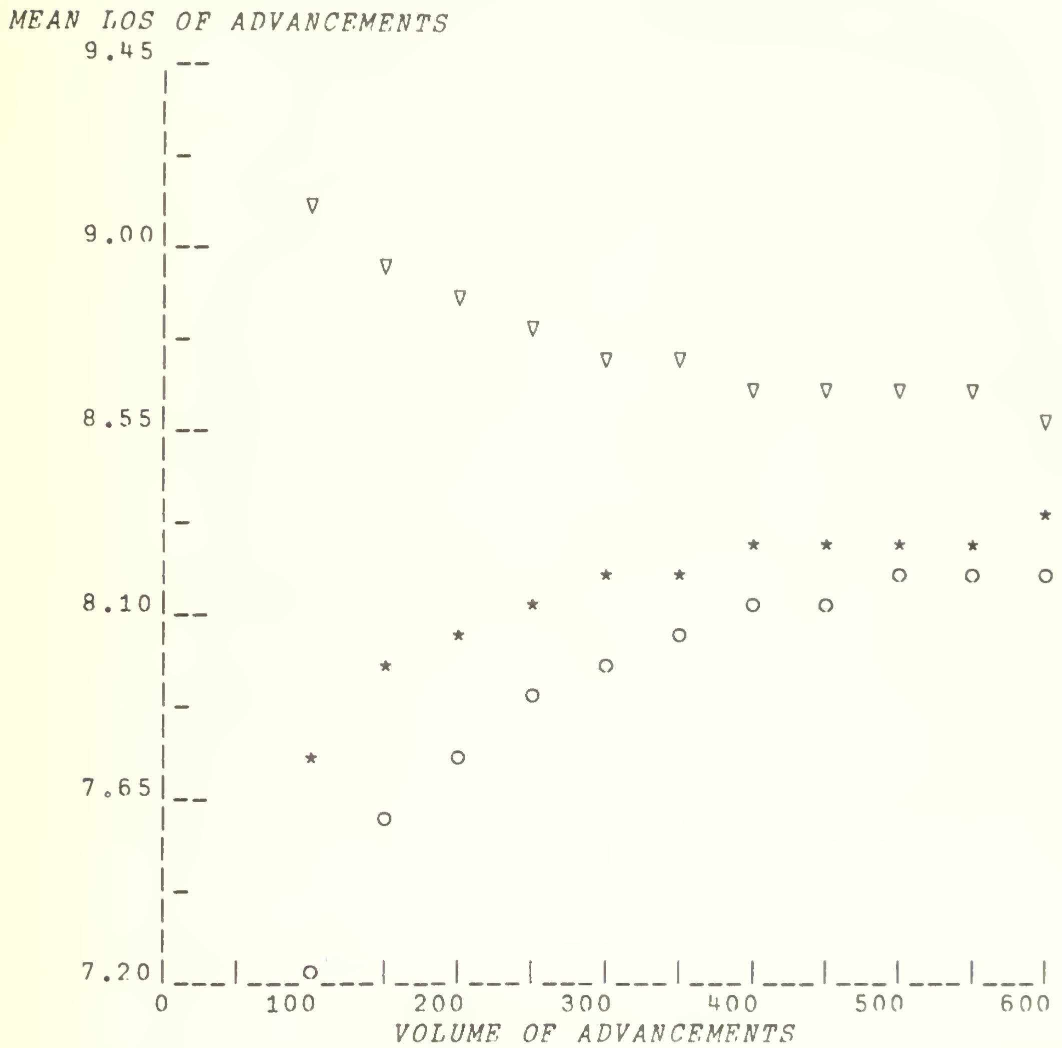


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	652	3.65	3.37
o 1969	1537	3.07	2.61
∇ 1966	938	3.56	4.16

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=300 *PAY GRADE*=6

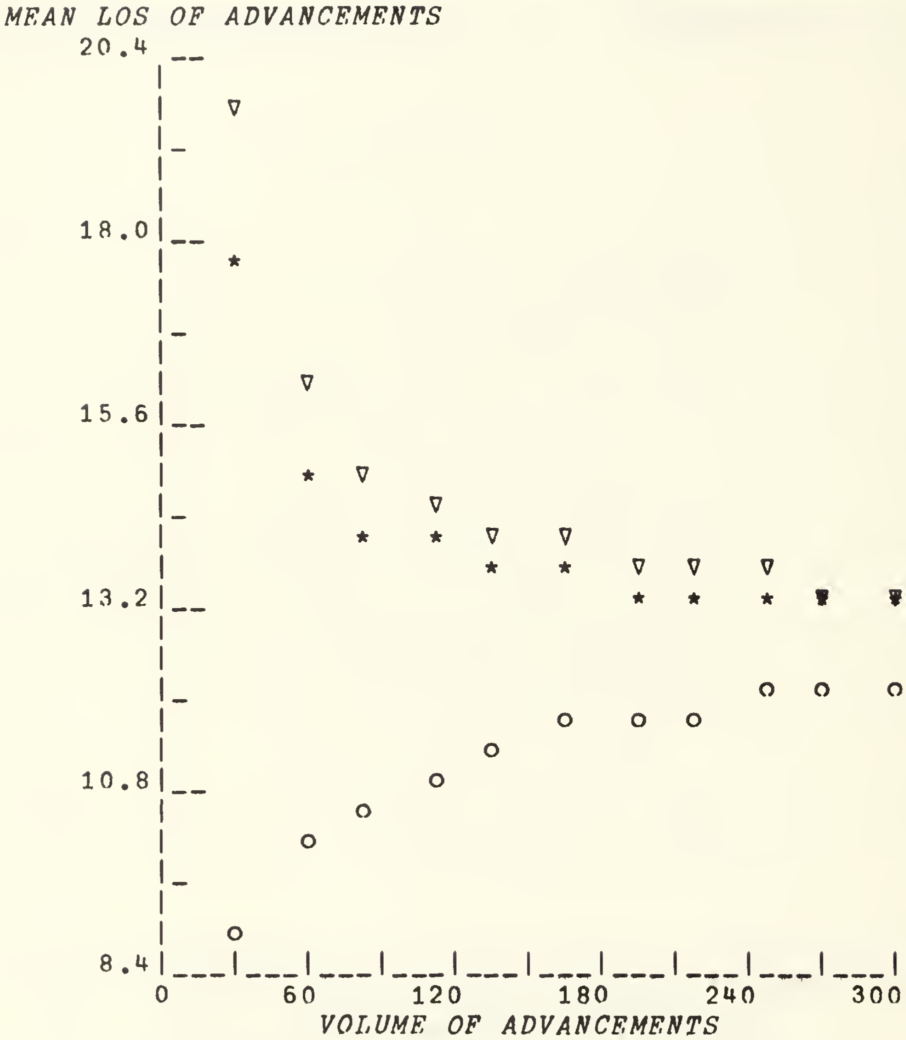


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
⋆ 1976	182	7.85	7.26
○ 1972	146	8.04	5.89
∇ 1966	158	7.22	9.69

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=300 PAY GRADE=7

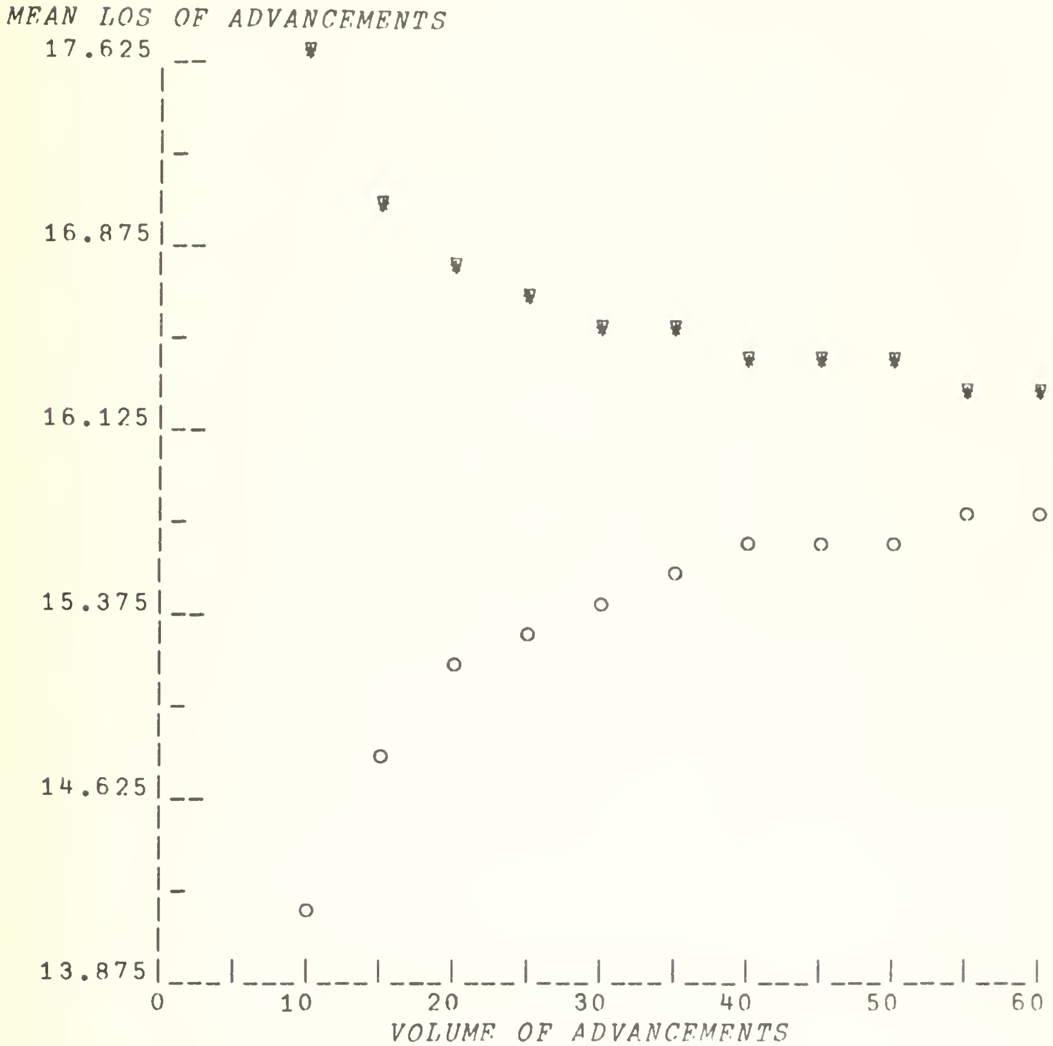


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
*	1976	113	14.44	12.40
o	1966	39	11.05	10.56
∇	1974	150	14.41	13.07

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=300 PAY GRADE=8

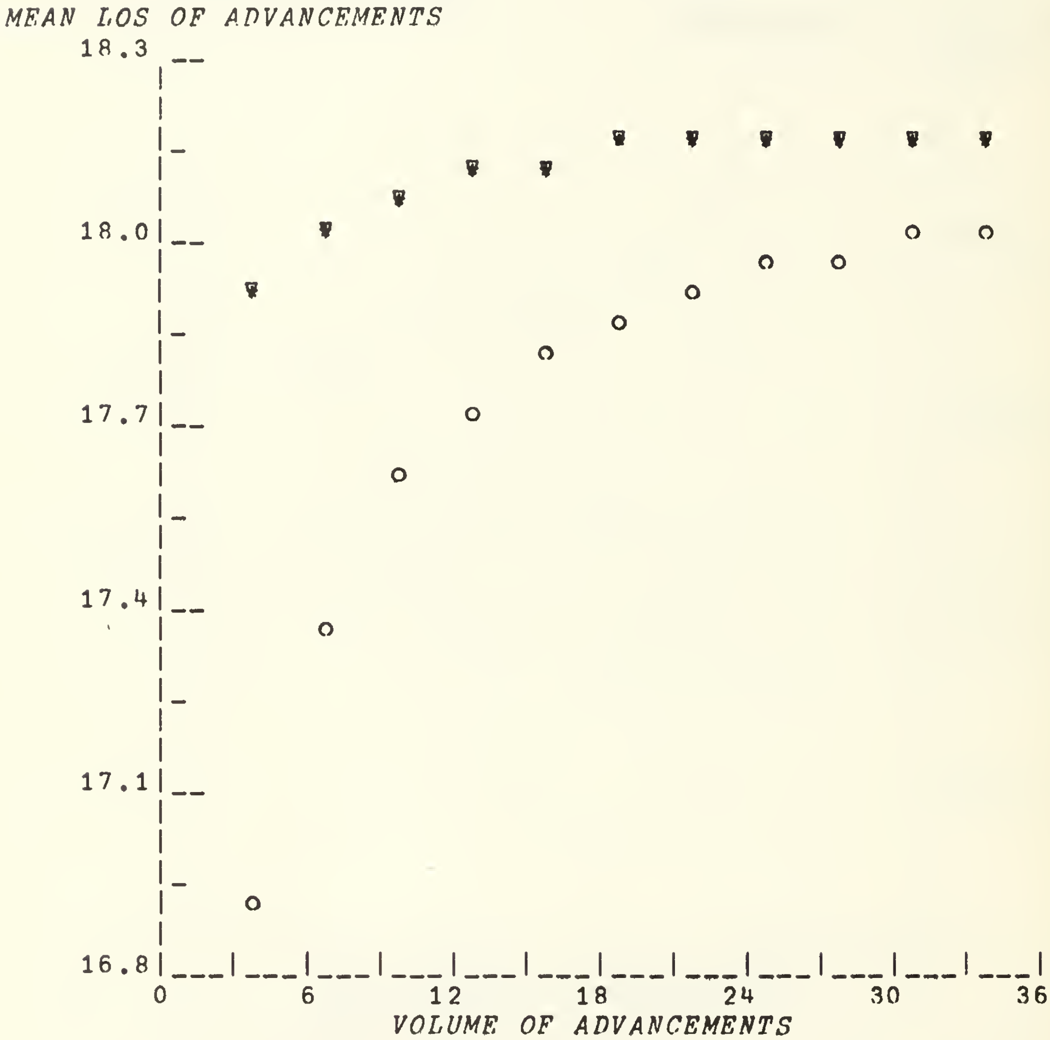


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
*	1976	19	16.37	17.18
o	1969	56	15.75	14.96
∇	1976	19	16.37	17.18

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=300 *PAY GRADE=9*

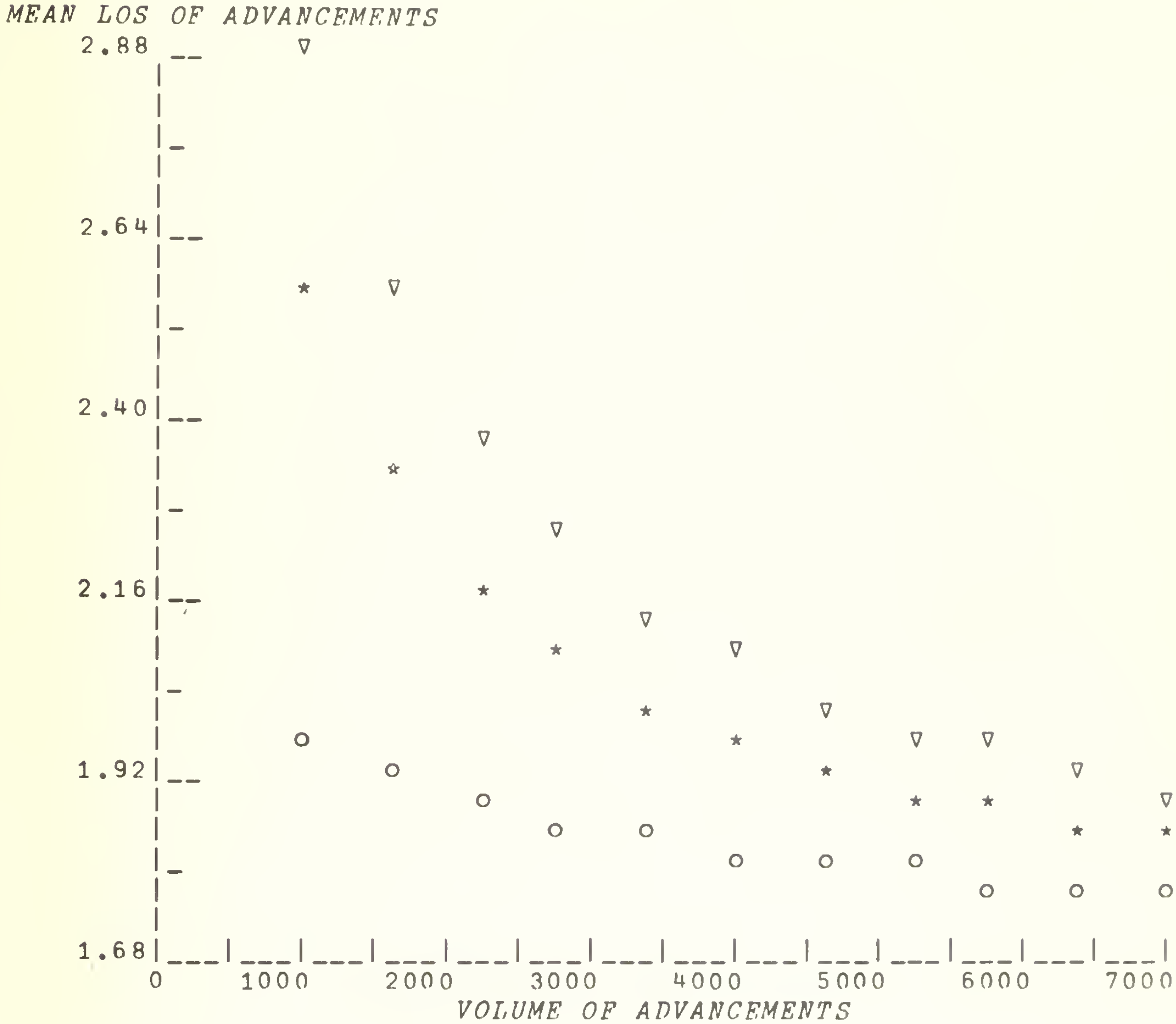


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
*	1976	6	18.50	18.95
o	1970	8	17.00	17.67
∇	1976	6	18.50	18.95

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 *PAY GRADE*=4

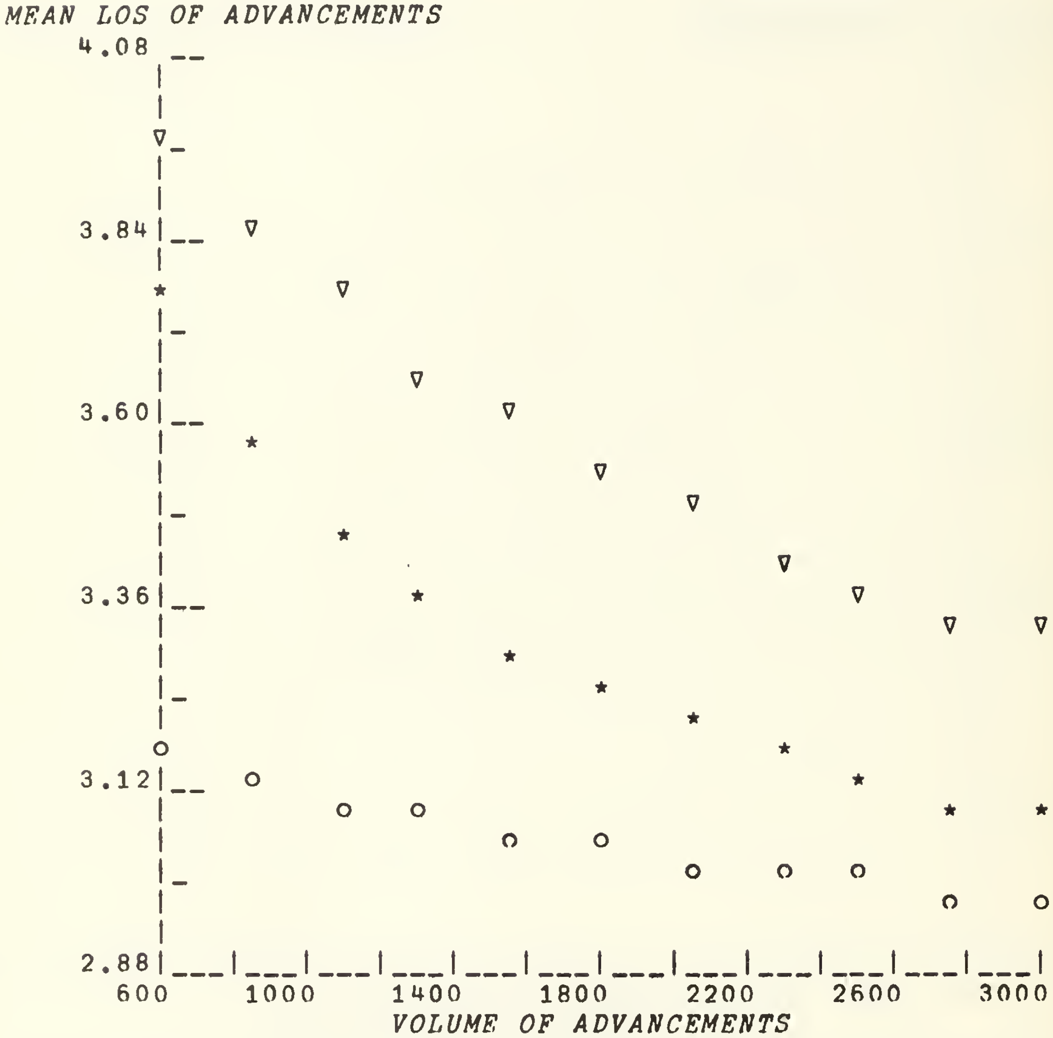


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	2247	2.28	1.73
o 1970	4701	1.71	1.33
∇ 1974	2157	2.18	1.88

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 *PAY GRADE*=5

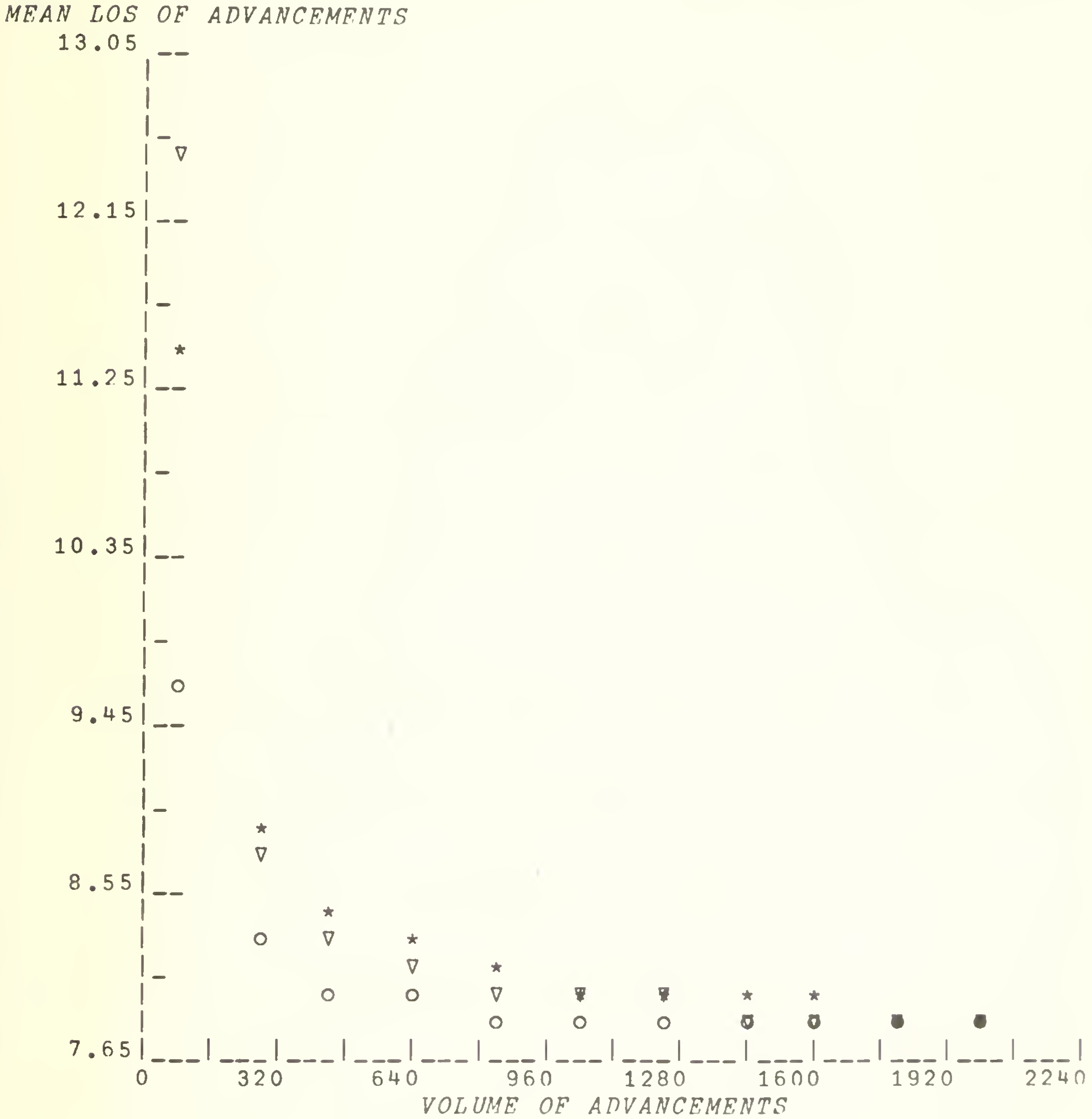


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
* 1976	608	4.44	3.49
o 1968	2382	3.40	2.90
∇ 1966	1978	3.54	4.05

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 *PAY GRADE*=6

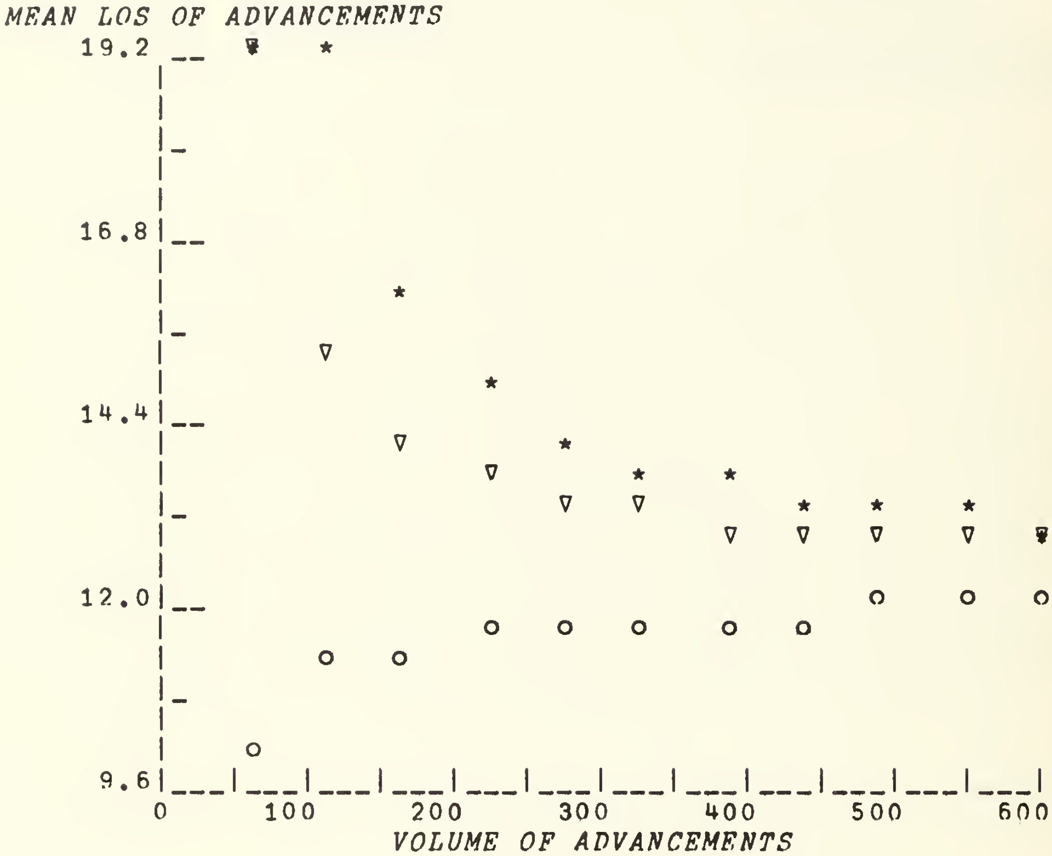


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
*	1976	150	10.20	7.95
o	1969	975	8.22	6.43
∇	1973	188	9.53	7.65

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 PAY GRADE=7

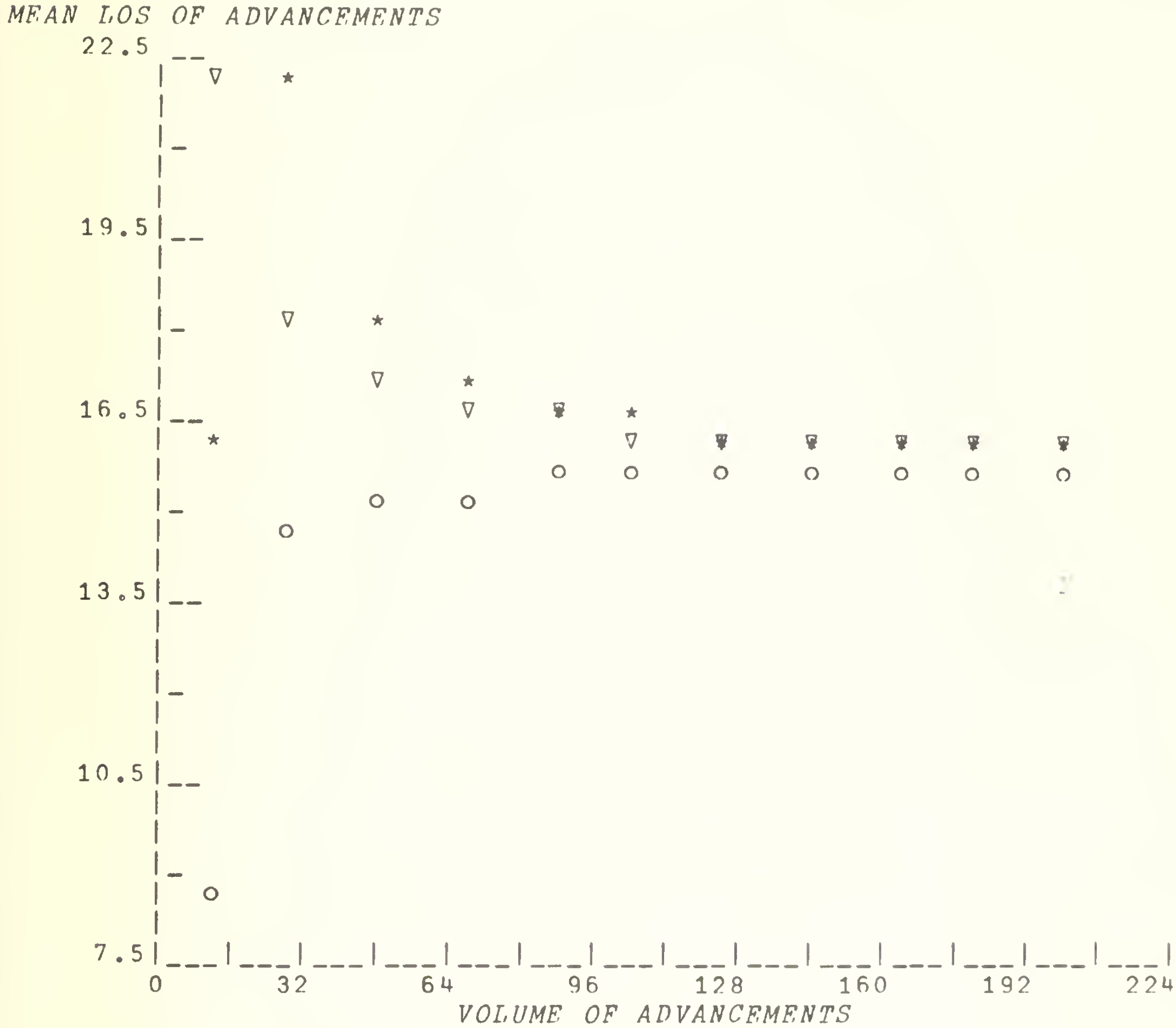


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	216	14.36	14.71
o 1966	316	11.91	11.19
∇ 1974	200	14.98	13.42

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 *PAY GRADE*=8

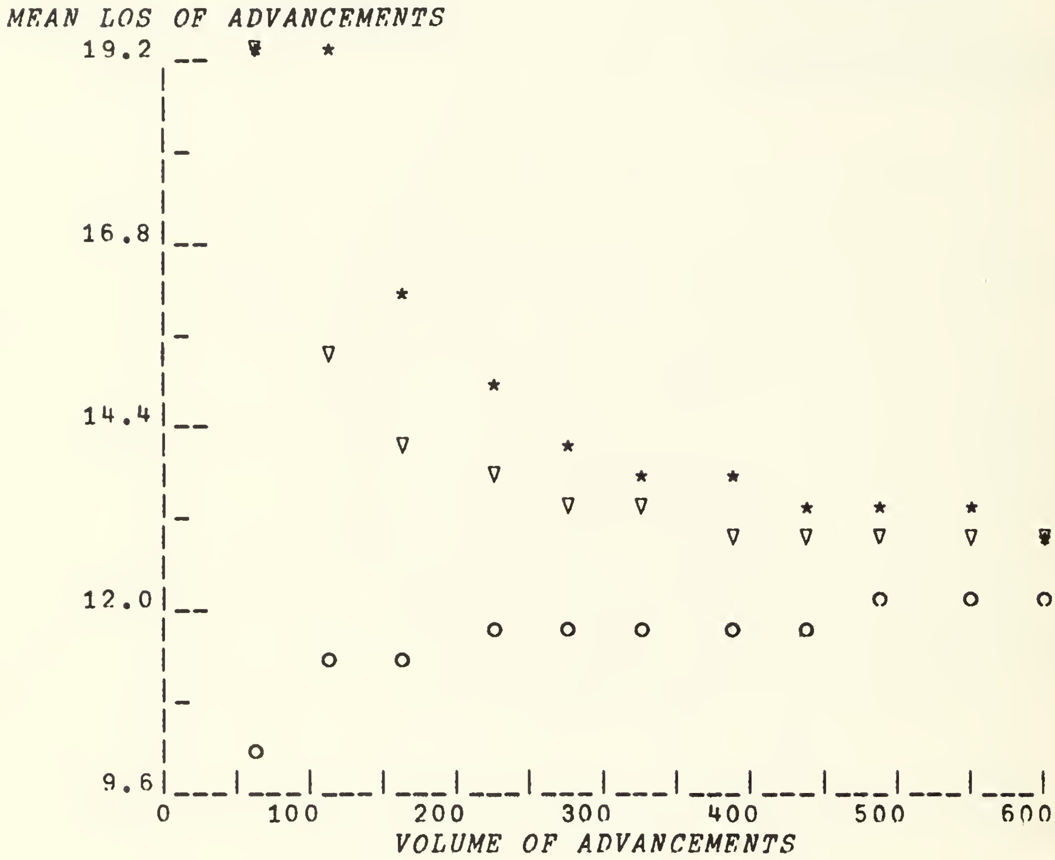


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
★ 1976	60	16.75	17.83
o 1967	162	14.23	15.93
∇ 1974	111	16.91	16.96

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 *PAY GRADE*=7

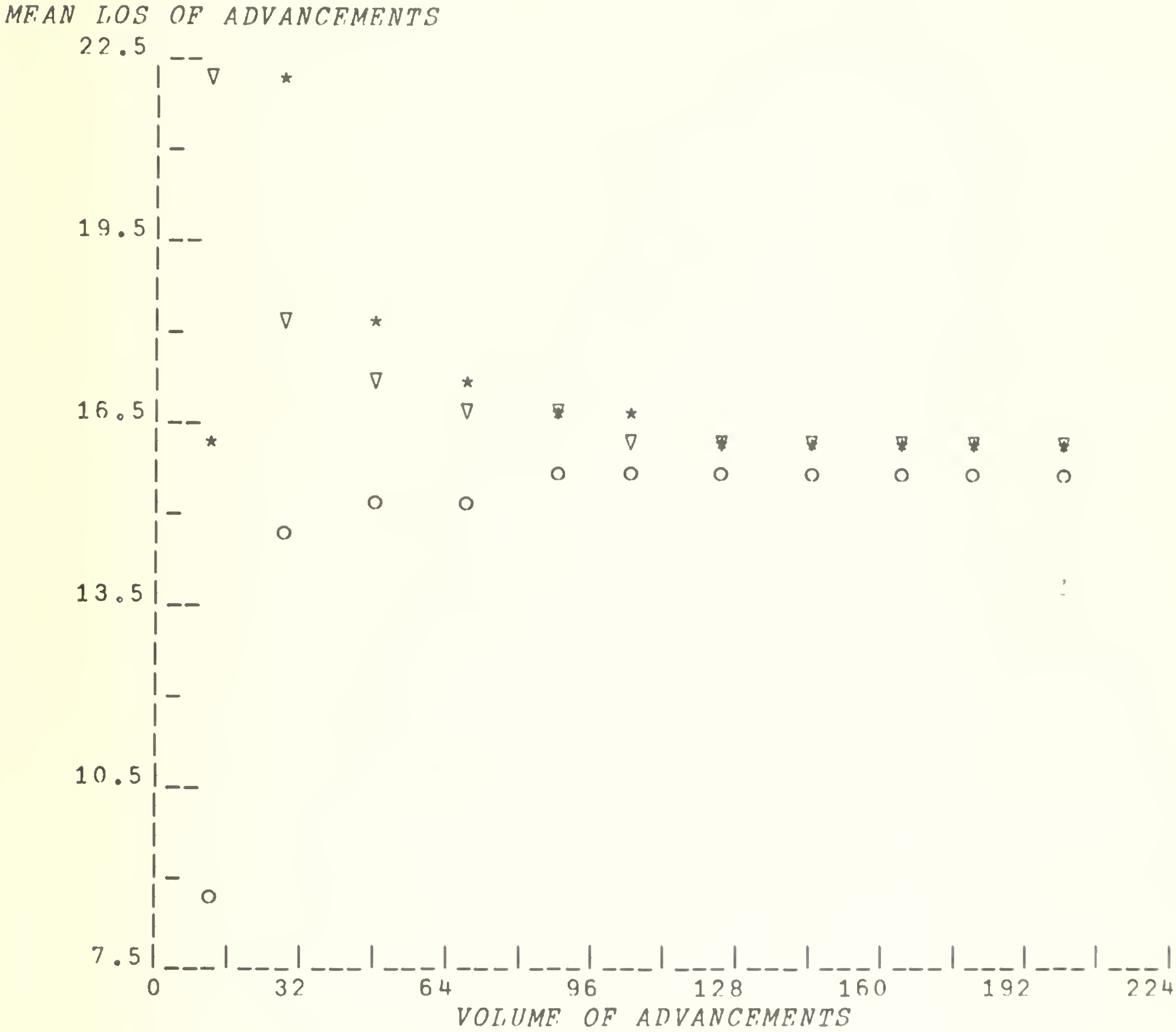


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
* 1976	216	14.36	14.71
o 1966	316	11.91	11.19
∇ 1974	200	14.98	13.42

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 *PAY GRADE*=8

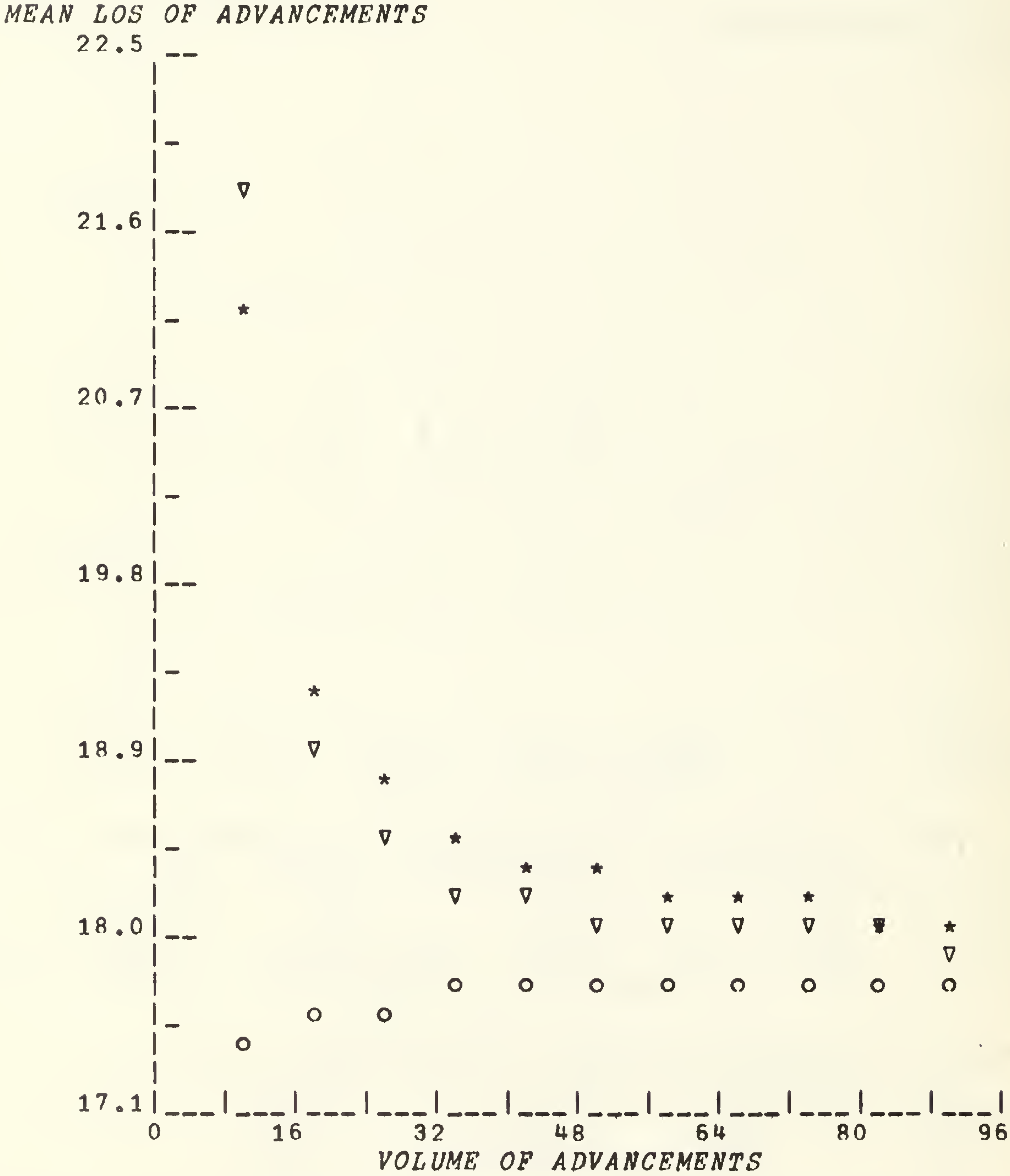


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
*	1976	60	16.75	17.83
o	1967	162	14.23	15.93
∇	1974	111	16.91	16.96

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1500 *PAY GRADE*=9

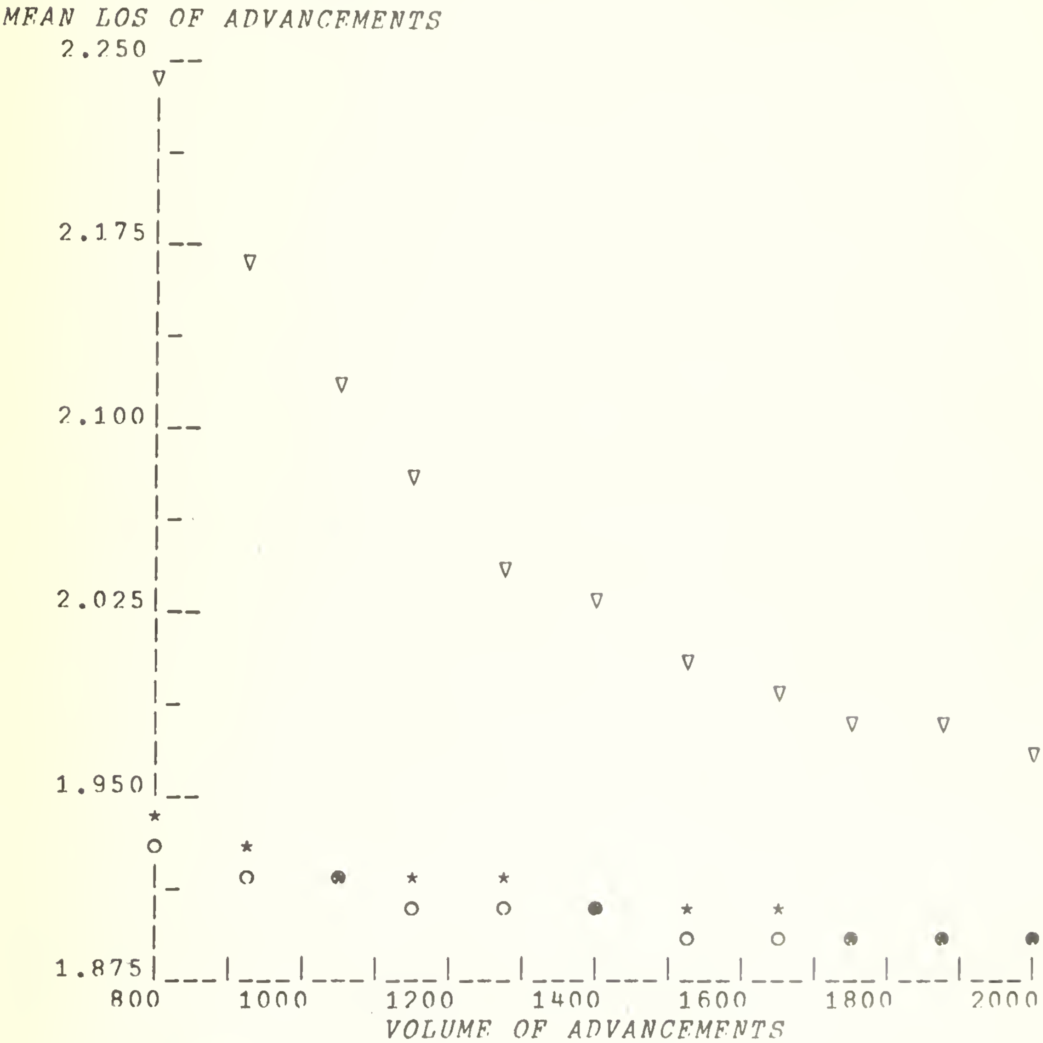


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
*	1976	21	18.90	19.52
o	1970	17	17.47	17.78
∇	1968	28	19.07	17.72

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1800 *PAY GRADE=4*

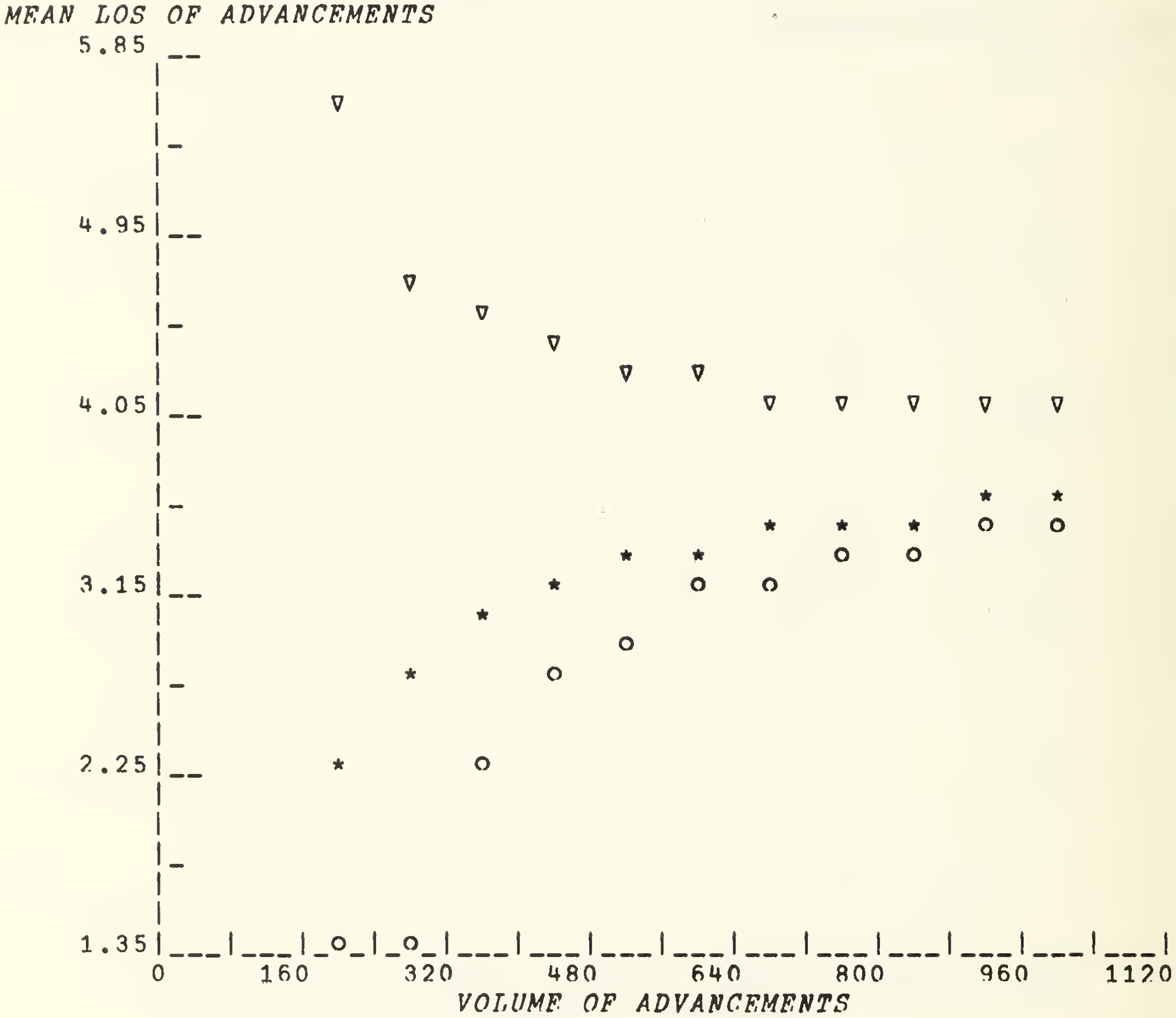


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
★ 1976	821	2.25	1.59
o 1975	1006	1.94	1.48
∇ 1968	1780	2.34	2.07

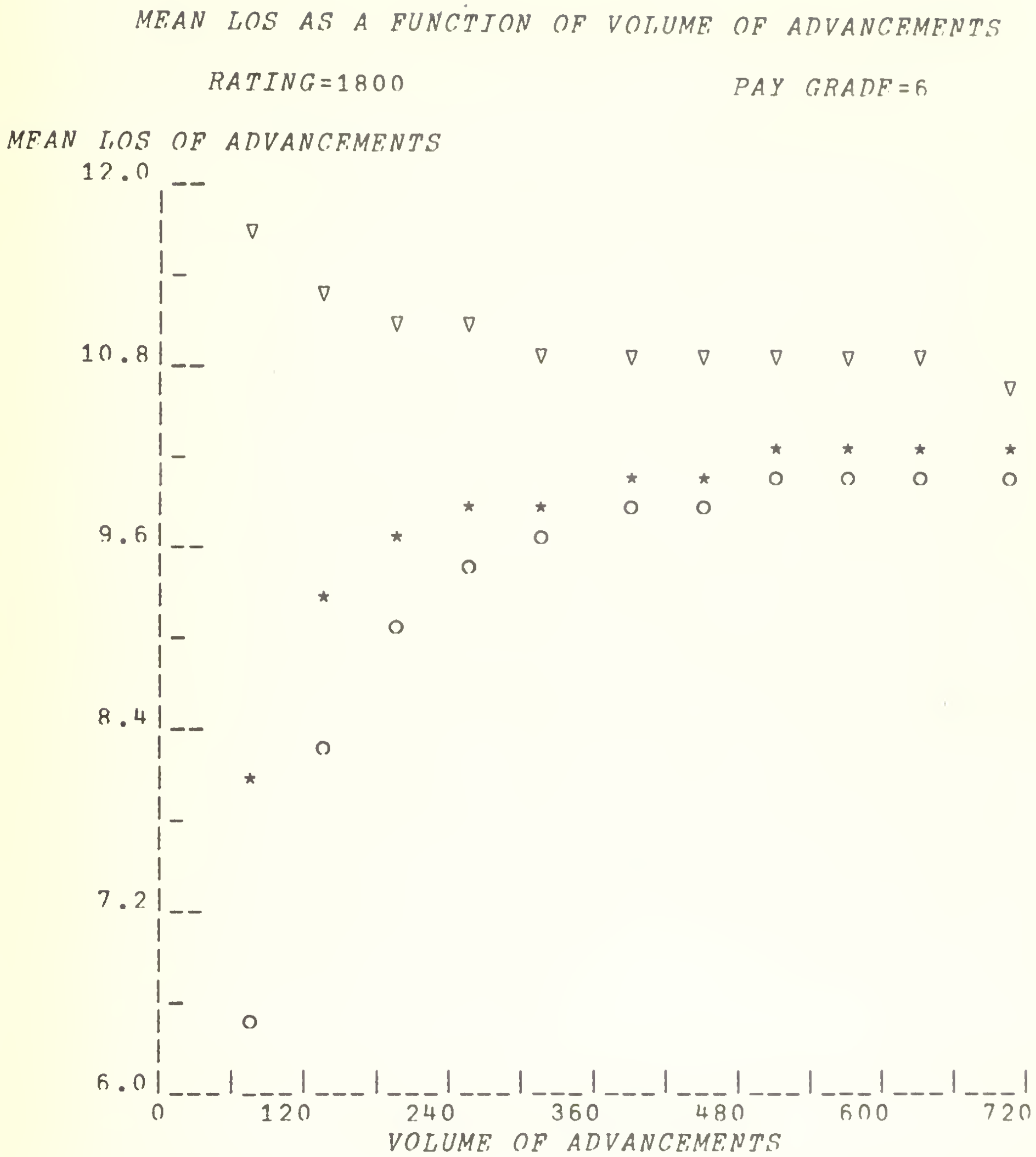
MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1800 PAY GRADE=5



FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
*	1976	704	3.77	3.45
o	1970	693	3.43	3.41
∇	1967	547	4.88	6.01

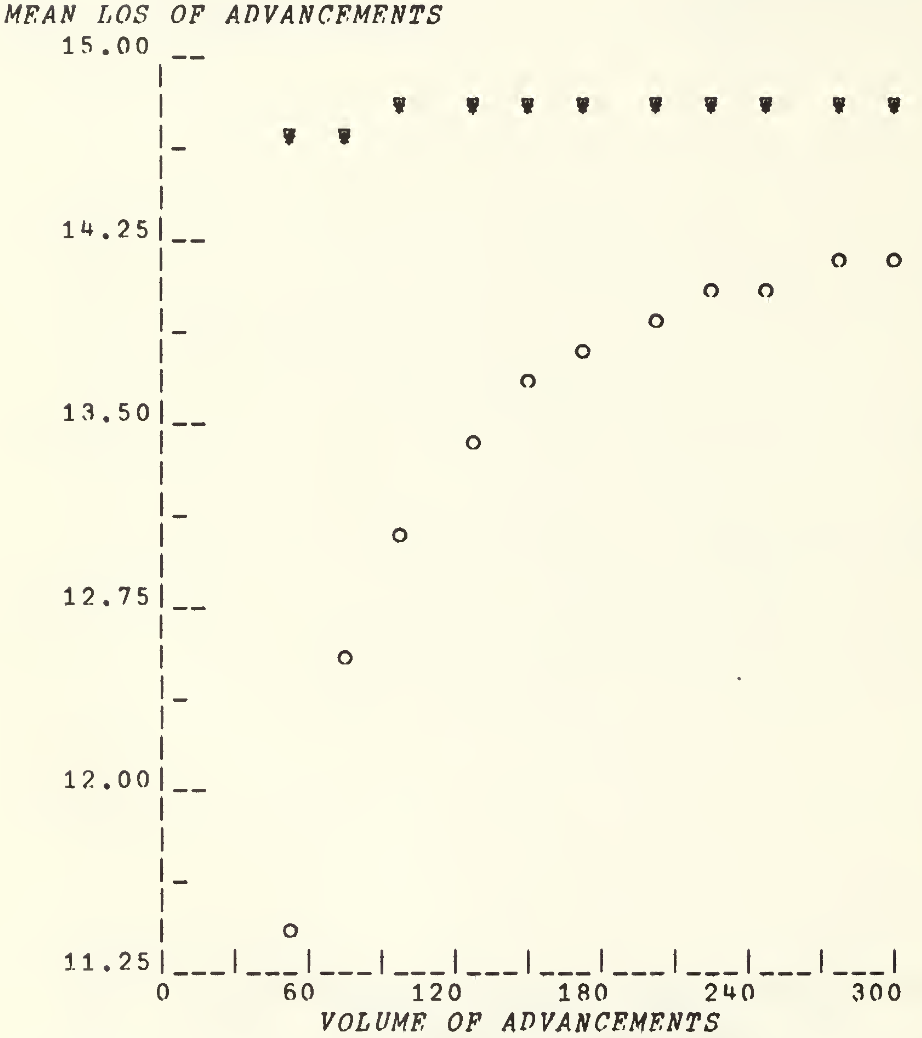


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
*	1976	314	8.93	7.28
o	1974	76	9.51	6.87
∇	1967	225	9.89	9.47

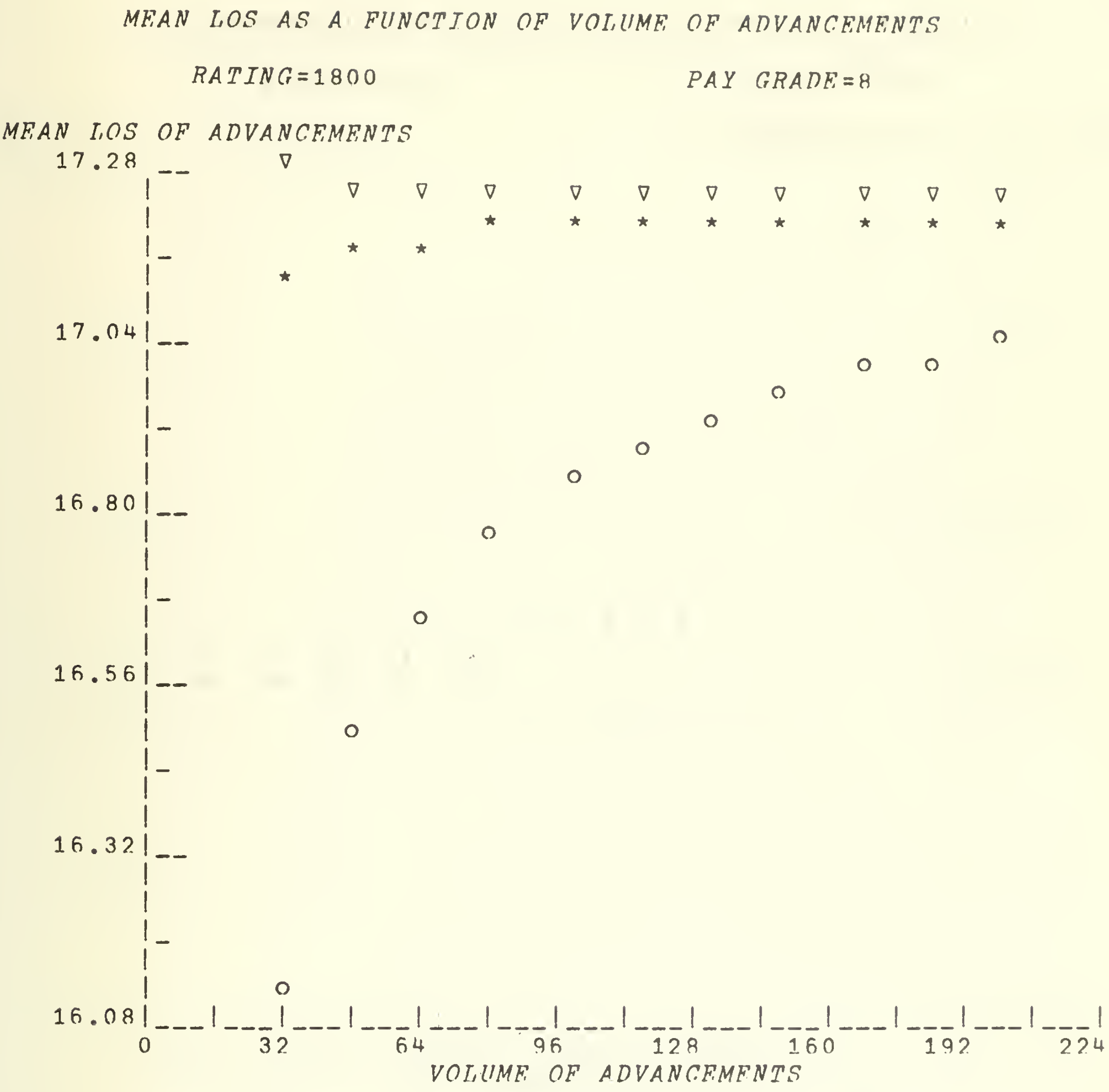
MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1800 *PAY GRADE*=7



FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
* 1976	181	14.10	13.91
o 1966	78	13.82	12.65
∇ 1976	181	14.10	13.91

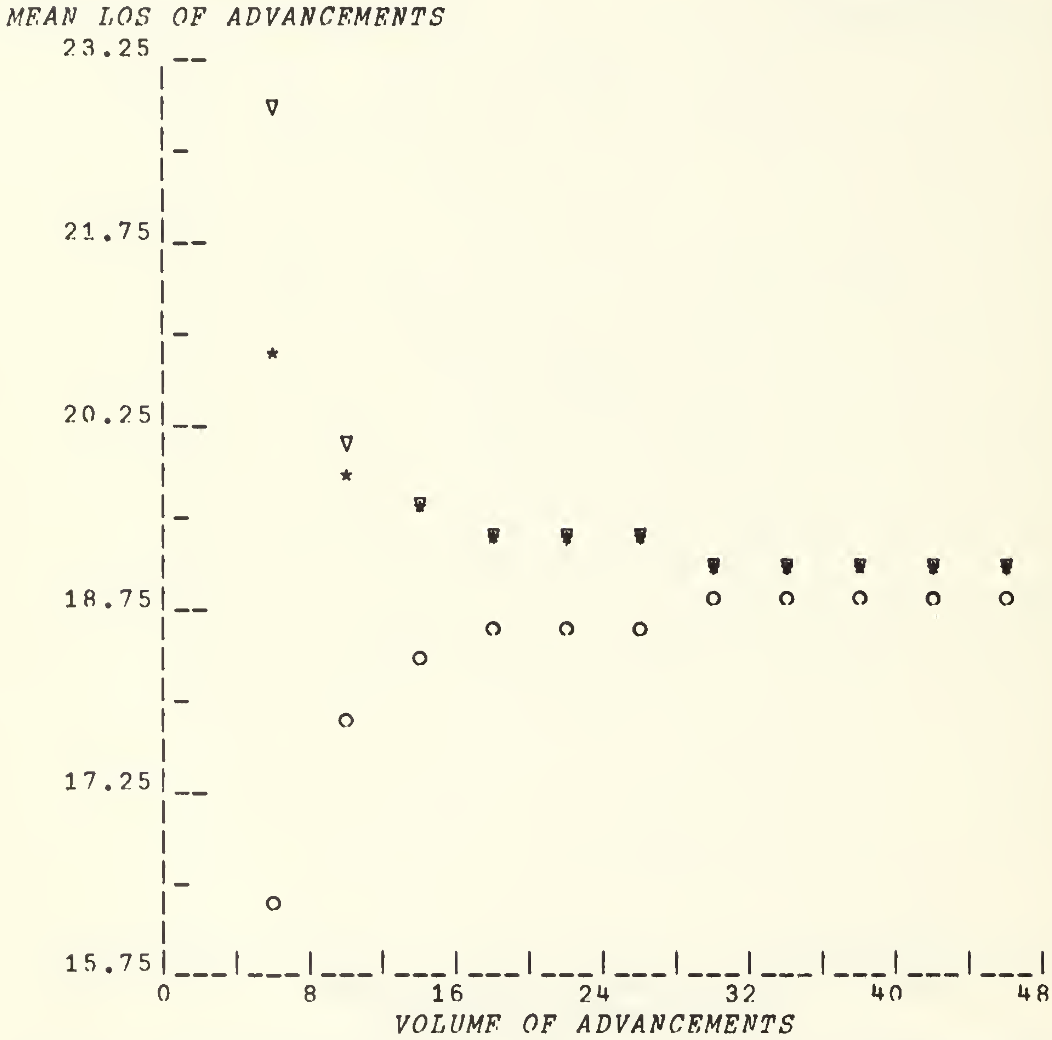


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	31	17.29	18.22
o 1969	65	18.20	15.94
∇ 1975	35	17.00	17.86

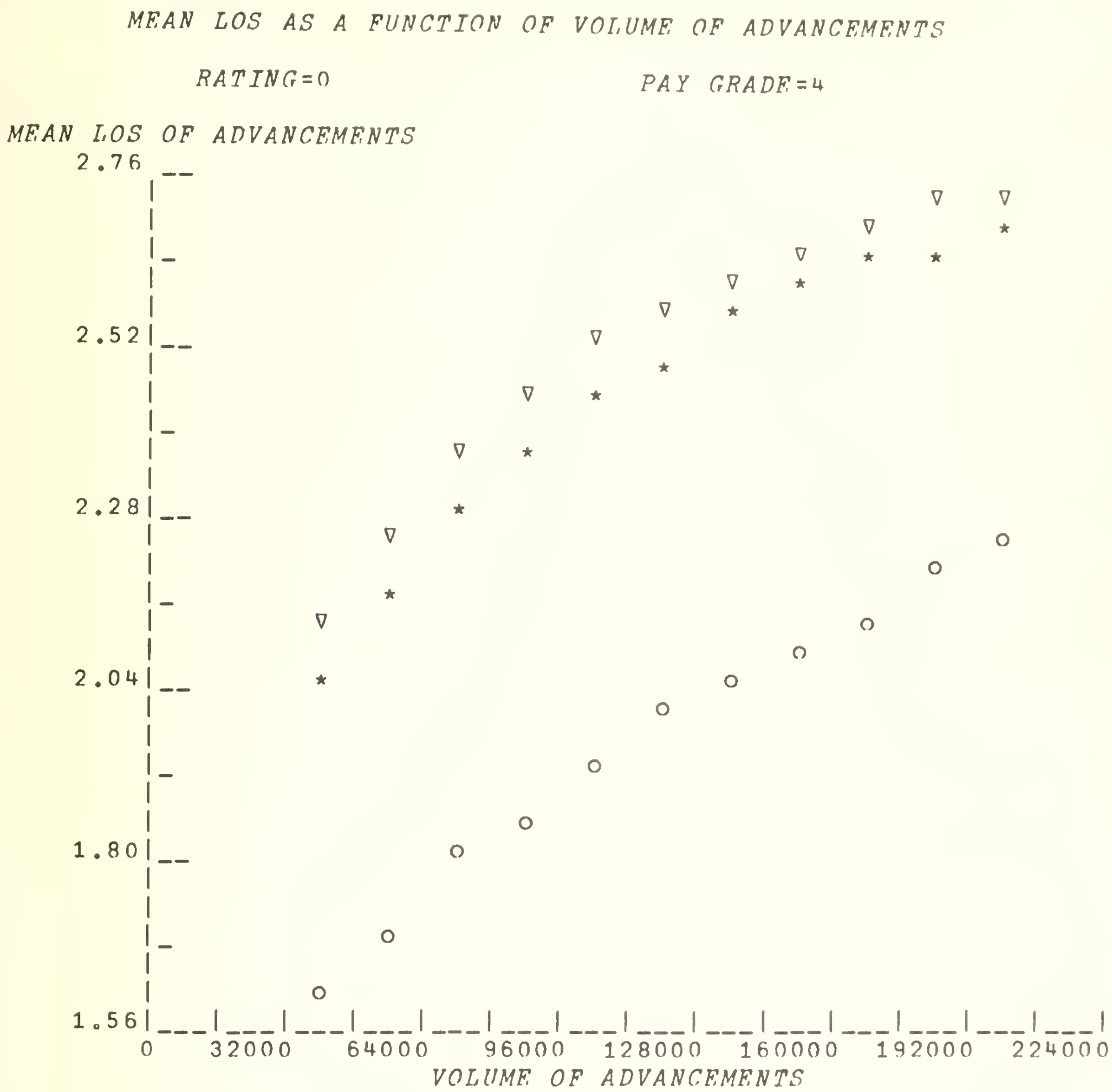
MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=1800 *PAY GRADE*=9



FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
* 1976	13	19.08	19.72
o 1966	23	18.78	19.09
∇ 1971	28	19.79	19.35

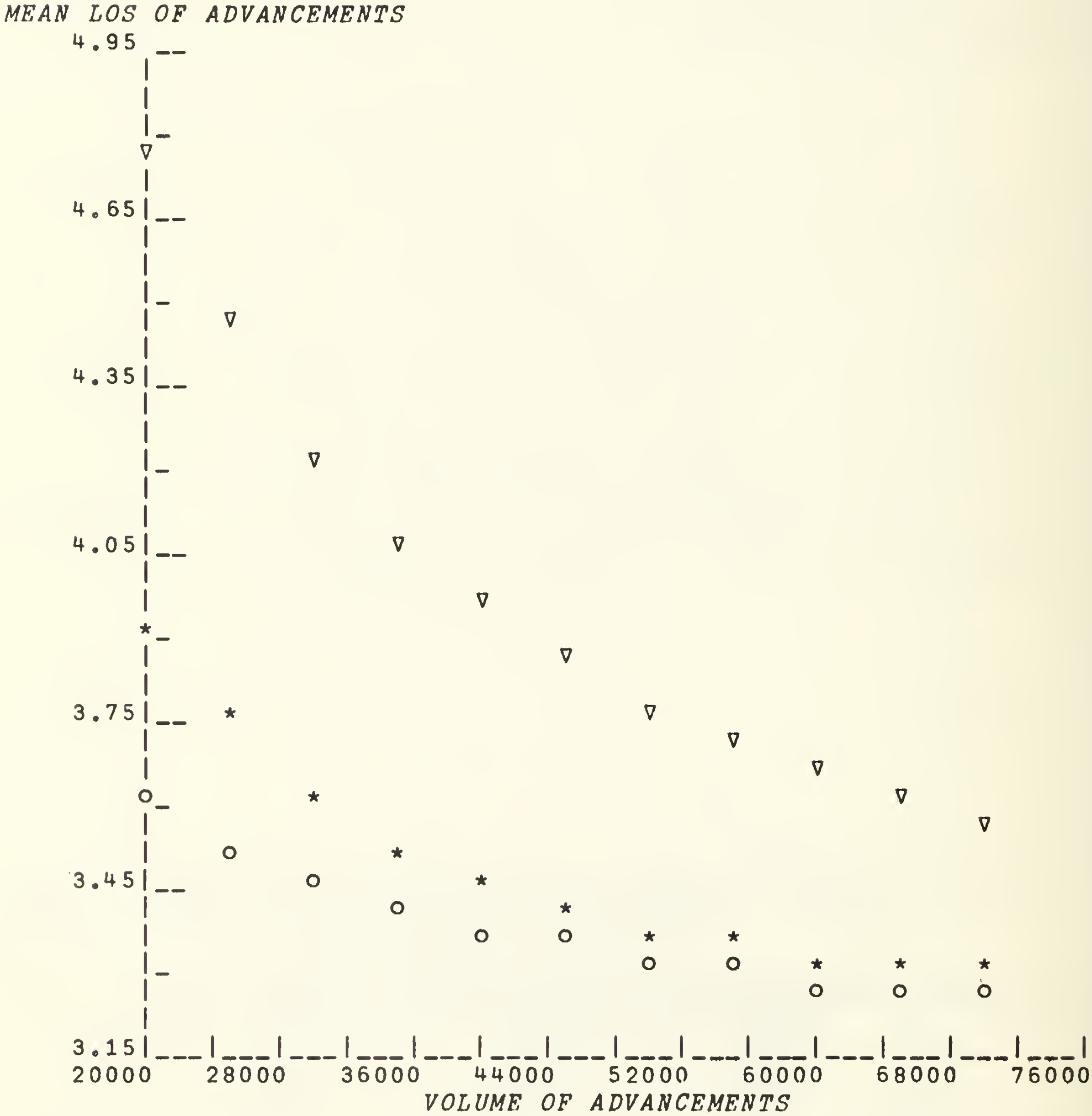


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
*	1976	49388	2.53	1.82
o	1967	94973	1.74	1.76
∇	1975	45183	2.33	1.88

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=0 *PAY GRADE*=5

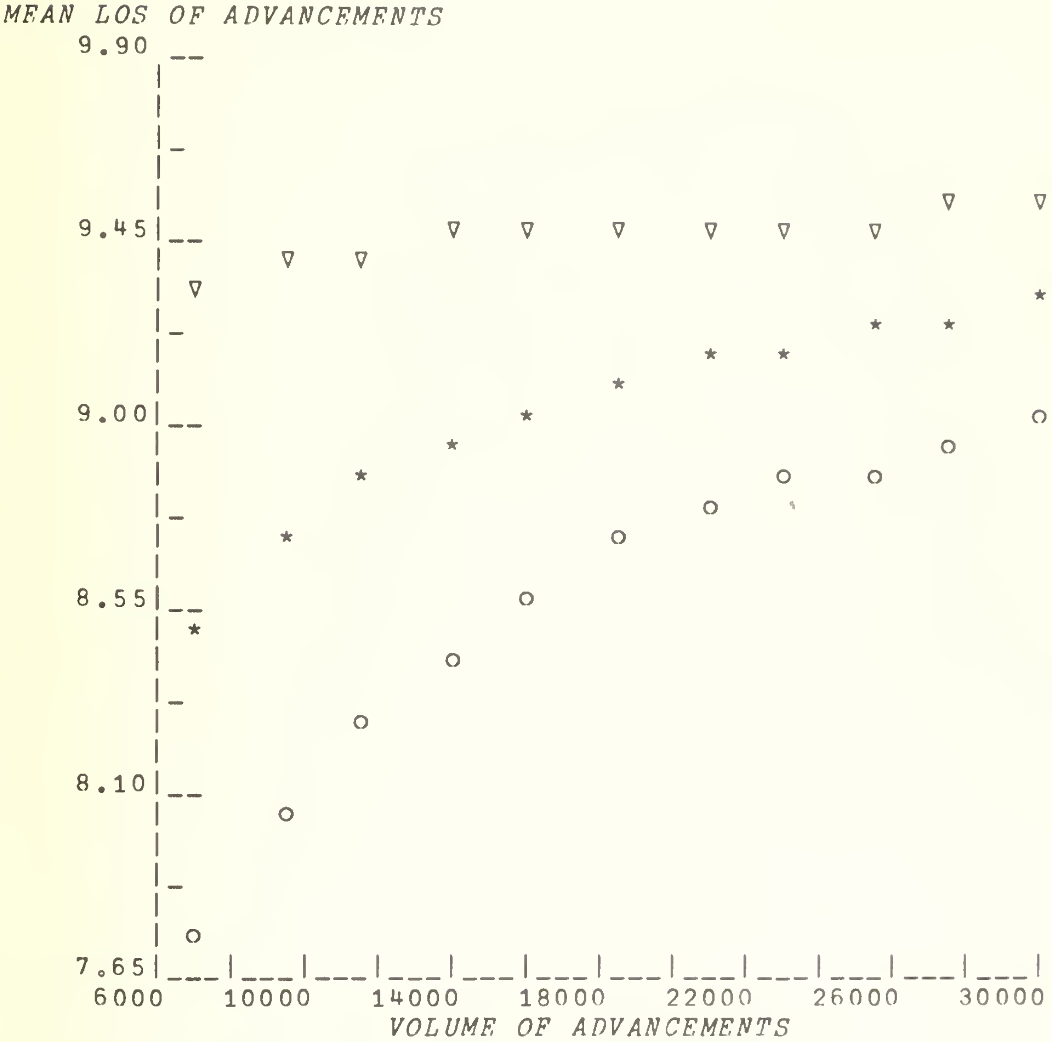


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	27166	4.52	3.70
o 1969	55337	3.34	3.13
∇ 1966	36919	4.40	5.16

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=0 *PAY GRADE=6*

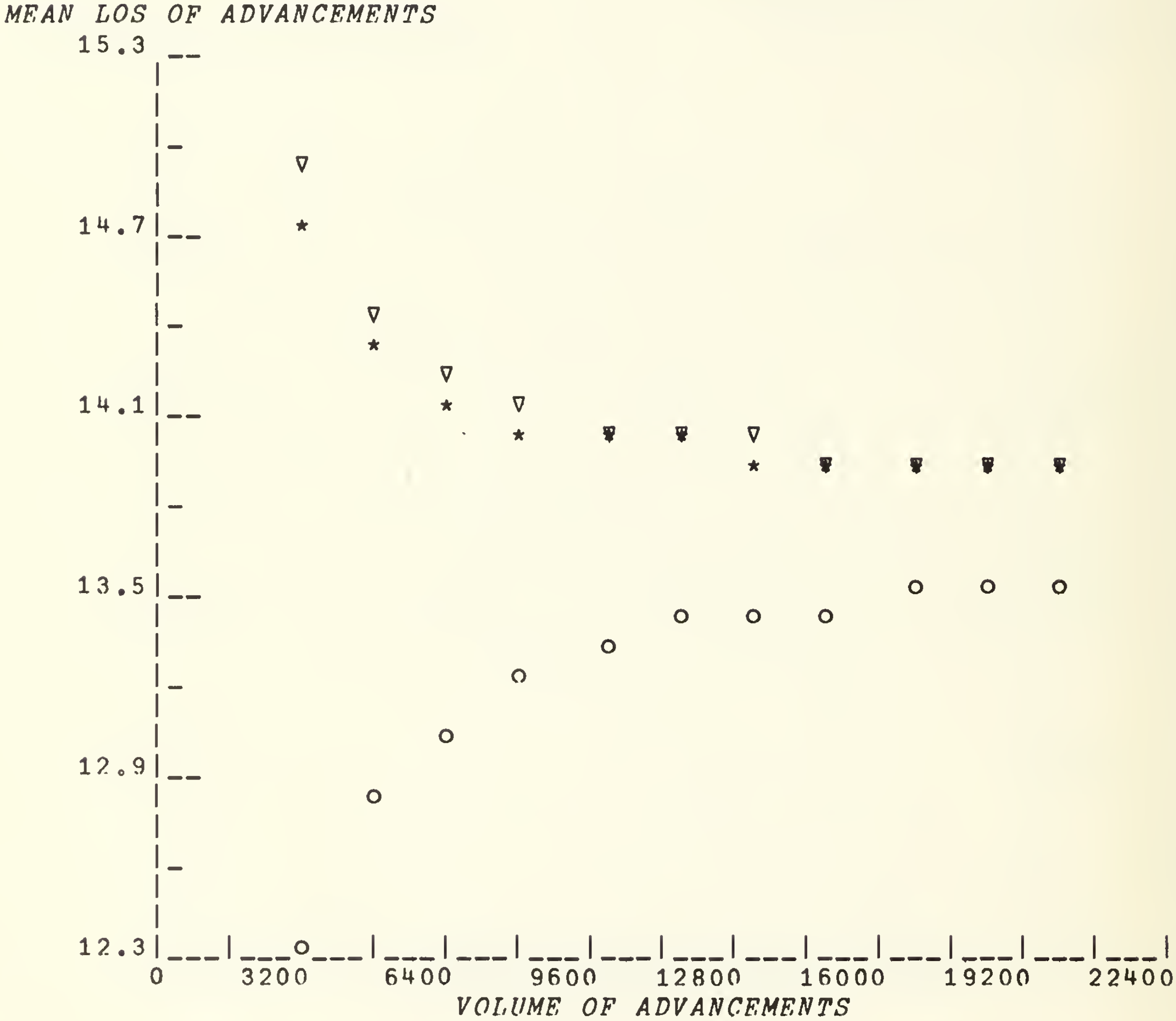


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
*	1976	12099	8.94	7.65
o	1969	19055	9.14	6.86
∇	1966	17433	9.08	9.10

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=0 *PAY GRADE=7*

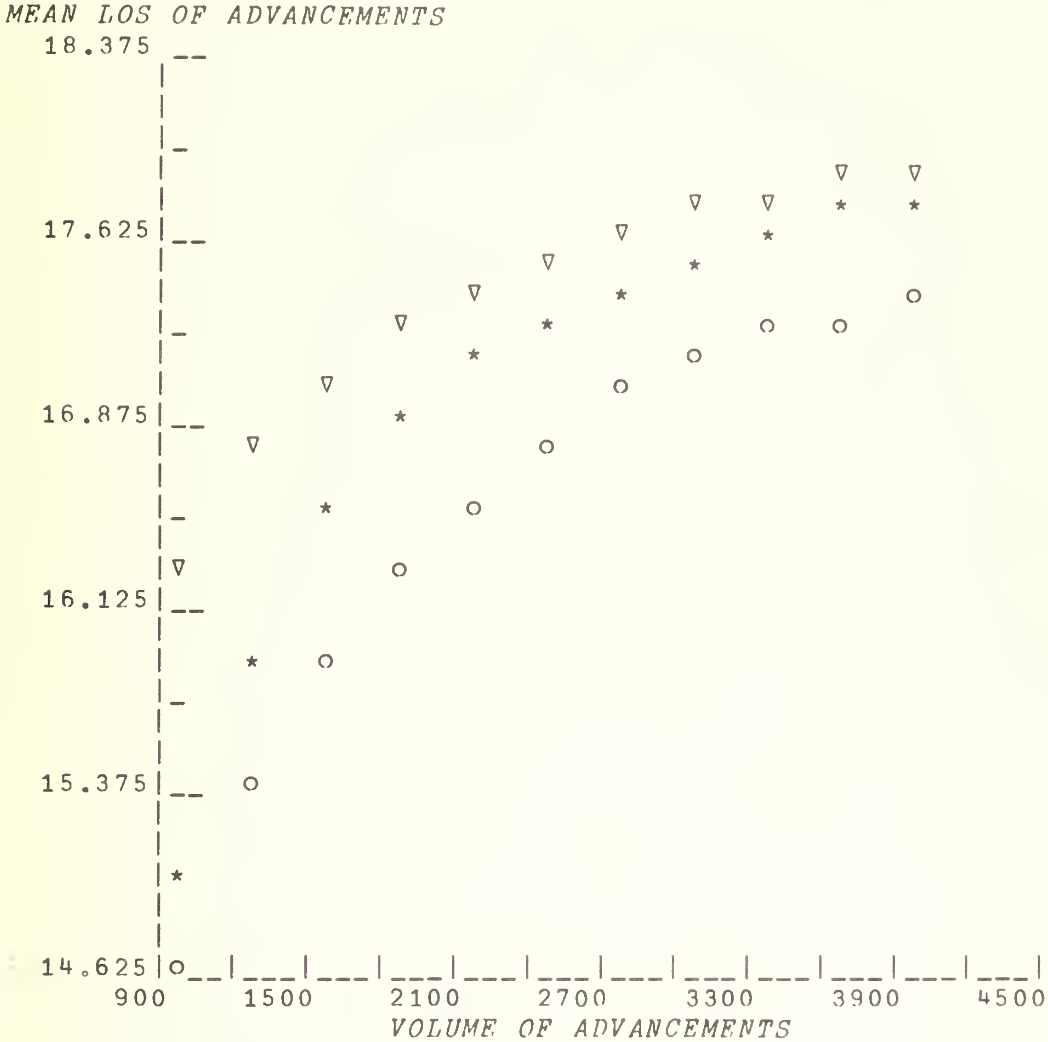


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

	FY	*VOLUME*	*ADV MEAN LOS*	*INV MEAN LOS*
*	1976	6790	14.31	13.16
O	1969	9105	13.88	11.79
∇	1975	4941	14.22	13.28

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=0 PAY GRADE=8

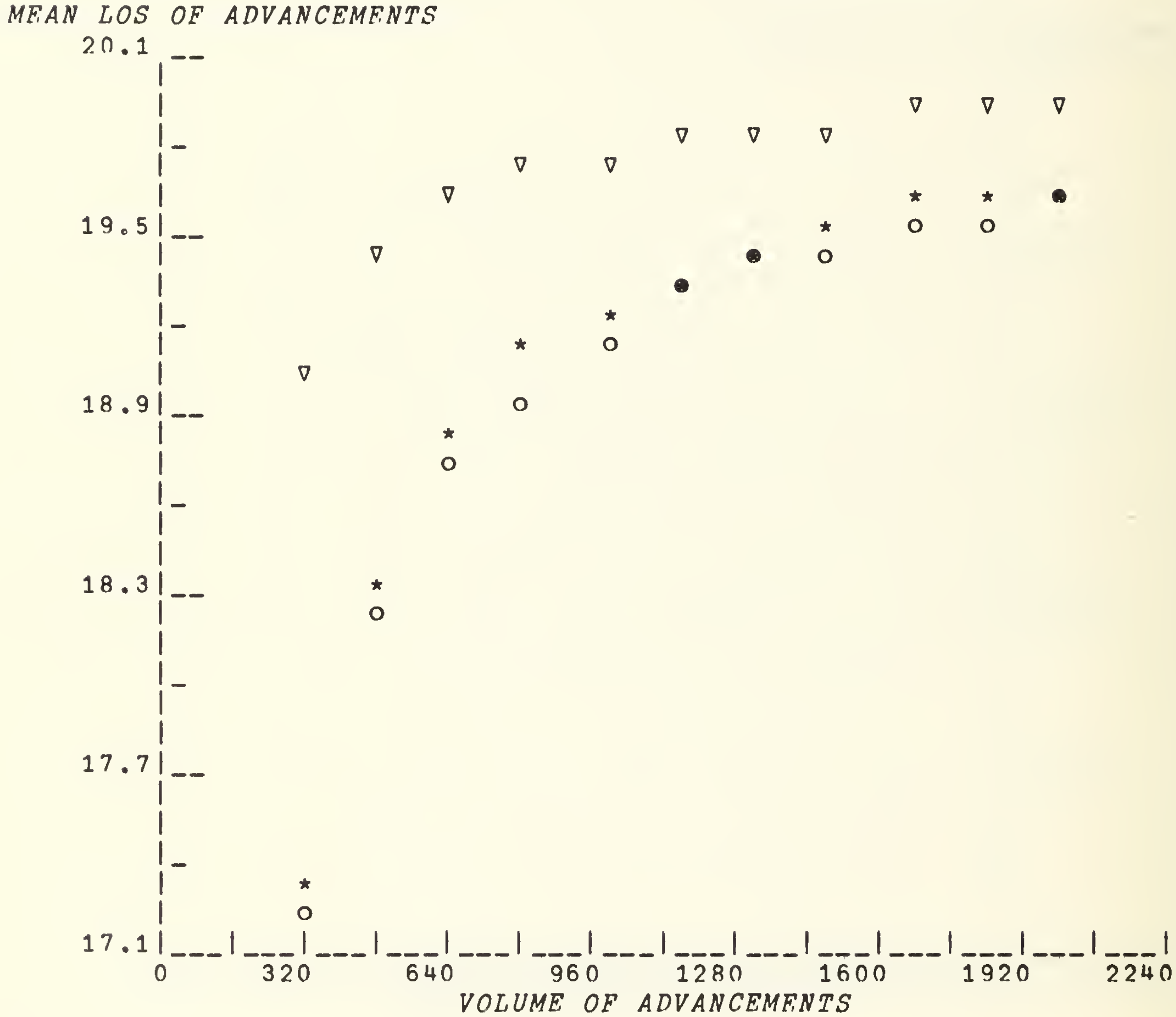


FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	1730	16.92	17.13
o 1970	2028	16.73	15.95
∇ 1966	1327	17.49	17.42

MEAN LOS AS A FUNCTION OF VOLUME OF ADVANCEMENTS

RATING=0 *PAY GRADE=9*



FY'S WHOSE INVENTORY LOS DISTRIBUTIONS ARE USED IN GRAPH:

FY	VOLUME	ADV MEAN LOS	INV MEAN LOS
* 1976	562	18.82	19.42
o 1975	603	18.52	19.20
∇ 1967	981	19.35	20.39

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